Lecture 24

NEGF continued

$$|\psi(t)\rangle = S(t,-\infty)\,|\psi(-\infty)\rangle$$

Recall, we turned the perturbation on adiabatically in the distant past. Thus at zero temperature,

$$|\psi(-\infty)\rangle = |\phi_0\rangle,$$ the ground state of H_0.

Furthermore:

$$|\psi(\infty)\rangle = S(\infty,0)\,|\psi(0)\rangle.$$

If we also turn off the interactions adiabatically in the far future, and we consider equilibrium systems, then $|\psi(\infty)\rangle \propto |\phi_0\rangle$.

e.g. $|\psi(\infty)\rangle = e^{i\theta}|\phi_0\rangle$

$$e^{i\theta} = \langle\phi_0| S(\infty,-\infty)|\phi_0\rangle.$$

Green's functions

$$G^t_{nm}(t_1,t_2) = -i\langle T\, d_n(t_1)\, d_m^\dagger(t_2)\rangle$$

Heisenberg representation

i) $t_1 > t_2$

$$G^t_{nm}(t_1,t_2) = -i\langle d_n(t_1)\, d_m^\dagger(t_2)\rangle$$
$$= G^>_{nm}(t_1,t_2)$$

ii) $t_2 > t_1$

$$G^t_{nm}(t_1,t_2) = +i\langle d_m^\dagger(t_2)\, d_n(t_1)\rangle$$
$$= G^<_{nm}(t_1,t_2)$$

converting to interaction rep.:

$$| \rangle = S(0,-\infty)\, |\phi_0\rangle$$ at zero temp. in equil.

$$d_n(t) = e^{iHt} e^{-iH_0 t} \hat{d}_n(t) e^{iH_0 t} e^{-iHt}$$

$$= U^{\dagger}(t)\, \hat{d}_n(t)\, U(t)$$

$$= S(0,t)\, \hat{d}_n(t)\, S(t,0)$$

$$G^{t}_{nm}(t_1,t_2) = -i\,\theta(t_1-t_2)\,\langle\phi_0|S(-\infty,0)$$
$$\times S(0,t_1)\,\hat{d}_n(t_1)\,S(t_1,0)\,S(0,t_2)\,\hat{d}^{\dagger}_m(t_2)$$
$$\times S(t_2,0)\,S(0,-\infty)\,|\phi_0\rangle$$

$$+ i\,\theta(t_2-t_1)\langle\phi_0|S(-\infty,0)\,S(0,t_2)\,\hat{d}^{\dagger}_m(t_2)\,S(t_2,0)$$
$$\times S(0,t_1)\,\hat{d}_n(t_1)\,S(t_1,0)\,S(0,-\infty)\,|\phi_0\rangle$$

$$\langle \phi_0 | S(-\infty, 0) = e^{-i\theta} \langle \phi_0 | S(\infty, -\infty) S(-\infty, 0)$$

$$= \frac{\langle \phi_0 | S(\infty, 0)}{\langle \phi_0 | S(\infty, -\infty) | \phi_0 \rangle}$$

$$\Rightarrow \quad G^{+}_{nm}(t_1, t_2) = \frac{-i}{\langle \phi_0 | S(\infty, -\infty) | \phi_0 \rangle}$$

$$\Big[\theta(t_1 - t_2) \langle \phi_0 | S(\infty, t_1) \hat{d}_n(t_1) S(t_1, t_2) \hat{d}^{+}_m(t_2) S(t_2, -\infty) | \phi_0 \rangle$$

$$- \theta(t_2 - t_1) \langle \phi_0 | S(\infty, t_2) \hat{d}^{+}_m(t_2) S(t_2, t_1) \hat{d}_n(t_1) S(t_1, -\infty) | \phi_0 \rangle \Big]$$

Finally,

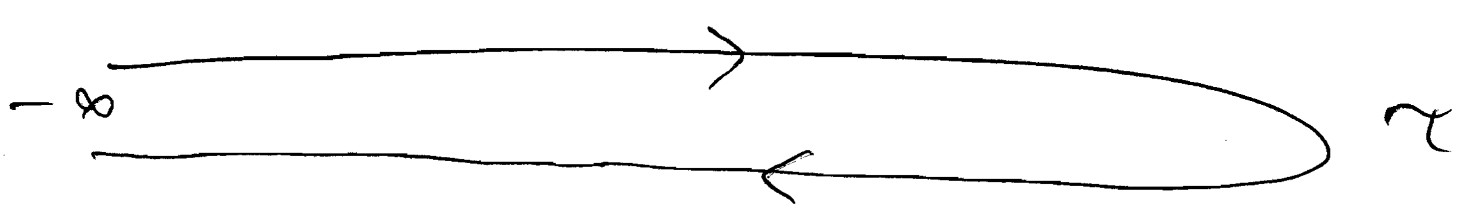
$$G^{t}_{nm}(t_1,t_2) = -i\,\frac{\langle \phi_0 | T\, \hat{d}_n(t_1)\, \hat{d}^{\dagger}_m(t_2)\, S(\infty,-\infty) | \phi_0 \rangle}{\langle \phi_0 | S(\infty,-\infty) | \phi_0 \rangle}$$

What happens out of equilibrium?

Particles, energy, etc. is transfered, entropy increases,

so $|\psi(\infty)\rangle \neq e^{i\theta} |\phi_0\rangle$

Keldysh time contour



Need two more GF:

$$G^r_{nm}(t_1, t_2) = -i\theta(t_1 - t_2)\left\langle \{ d_n(t_1), d_m^\dagger(t_2) \} \right\rangle$$

$$G^a_{nm}(t_1, t_2) = i\theta(t_2 - t_1)\left\langle \{ d_n(t_1), d_m^\dagger(t_2) \} \right\rangle$$

$$G^a - G^r = G^< - G^>$$