Problem Set 3; Due Friday, September 24

1) Marder 13.4

In (b), show more generally that

$$[a_{\mathbf{k}\nu}, a^{\dagger}_{\mathbf{k}'\nu'}] = \delta_{\mathbf{k}\mathbf{k}'}\delta_{\nu\nu'}.$$

2) Two-dimensional Debye model

Repeat the steps in the derivation of the Debye model for a two-dimensional crystal, assuming there are only two acoustic modes (one longitudinal and one transverse in-plane) with frequencies

$$\omega_s(\mathbf{k}) = v|\mathbf{k}|, \quad s = 1, 2,$$

where v is the speed of sound.

a) Determine the Debye frequency and the phonon density of states.

b) Write down a general expression for the thermal average energy of the system.

c) Show that the specific heat obeys a T^2 law in two dimensions.

3) Thermal fluctuations of a 2D crystal

Using Eq. (13.38a), calculate the mean-square displacement of an atom in the two-dimensional crystal of problem 2. Show that $\langle u_\ell^2 \rangle \to \infty$ for $T > 0$.