Exercises for Physics 560A

Problem Set 7; Due November 13

1) Conductance of a perfect 2D wire

Using the results for the electrical conductance given in Lecture 5, determine the resistance in Ohms of a perfect two-dimensional wire in the form of a strip of width $D = 3.75\lambda_F$, where λ_F is the de Broglie wavelength of an electron at the Fermi energy. Assume the conduction electrons in the wire can be described by the free-particle Schrödinger equation with Dirichlet boundary conditions (i.e., $\Psi = 0$) along the edges of the strip.

2) Sharvin formula

The Sharvin formula for the electrical conductance of an extremely short contact of area A between two pieces of metal is

$$G \simeq \frac{2e^2}{h}\frac{k_F^2 A}{4\pi}.$$

Derive the Sharvin formula by considering the total current flowing through a hole of area A in a thin insulating barrier separating two free electron gases with different Fermi energies. Use purely macroscopic arguments. Hint: In a free electron gas, the number of electrons with energy between E and $E+dE$ traveling at an angle between θ and $\theta + d\theta$ with respect to a given axis is

$$\frac{\partial^2 n}{\partial E \partial \theta} dE d\theta = \frac{D(E)}{2} \sin\theta \, dE \, d\theta,$$

where $D(E)$ is the density of states.

3) Solution of Schrödinger equation inside a cylinder

Verify that the wavefunction

$$\psi_{kmn}(r, \phi, z) = e^{ikz+im\phi} J_m(\gamma_{mn} r/R)$$

is a solution of the free-particle Schrödinger equation in cylindrical coordinates, with energy eigenvalue

$$E = \frac{\hbar^2 \gamma_{mn}^2}{2mR^2} + \frac{\hbar^2 k^2}{2m}.$$

This wavefunction vanishes on the surface of a cylinder of radius R if γ_{mn} is the nth zero of J_m.