Extra Credit Problem for Physics 560A

due in class Friday October 22

Equilibrium Fluctuations: Fermions and Bosons

Show that in thermal equilibrium the mean-square fluctuations in the occupancy of a single quantum state of energy ε is

$$\langle \Delta n^2 \rangle = f_\pm(\varepsilon)[1 \mp f_\pm(\varepsilon)],$$

where

$$f_\pm(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)} \pm 1}$$

are the Fermi-Dirac (+) and Bose-Einstein (-) distributions, respectively. Note that $\langle \Delta n^2 \rangle \equiv \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2$, where $\hat{n} = a^\dagger a$ and $aa^\dagger \pm a^\dagger a = 1$.