

Exercises for Physics 560A

Problem Set 4; Due Friday, September 30

1) Verify that the total number operator

$$\hat{N} = \sum_{\nu} a_{\nu}^{\dagger} a_{\nu}$$

for a system of bosons commutes with a product of creation and annihilation operators if and only if the number of a^{\dagger} 's equals the number of a 's in the product.

b) Repeat the derivation for fermions.

2) Two-body interaction

Verify that the expectation value of the 2-fermion interaction

$$H^{(2)} = \frac{1}{2} \sum_{ijklm} V_{ij\ell m} c_j^{\dagger} c_m^{\dagger} c_{\ell} c_i$$

in the state $|\mu\nu\rangle = c_{\mu}^{\dagger} c_{\nu}^{\dagger} |0\rangle$ is

$$\langle\mu\nu|H^{(2)}|\mu\nu\rangle = V_{\mu\mu\nu\nu} - V_{\mu\nu\nu\mu}.$$

Here use has been made of the symmetry $V_{ij\ell m} = V_{\ell mij}$.