

Exercises for Physics 560A

Problem Set 5; Due Friday, October 14

Two-dimensional electron gas

Consider a system of N non-interacting electrons in a two-dimensional box in the form of a square of side L , with area $A = L^2$. For simplicity, you may impose periodic boundary conditions.

a) Show that the density of states is independent of energy, and is given by

$$D(E) = \frac{mA}{\pi\hbar^2} = \frac{N}{E_F},$$

where E_F is the energy of the highest occupied state at $T = 0$.

b) Show that the chemical potential is given by

$$\mu = \beta^{-1} \ln(e^{\beta E_F} - 1),$$

where $\beta = 1/k_B T$.

c) Show that for temperatures $T \ll E_F/k_B$, the specific heat is

$$C_V \simeq \frac{\pi^2}{3} N k_B \frac{k_B T}{E_F}.$$