Physics 560A Problem Set 7; Due November 4

Donor States in Silicon

A single pentavalent atom (impurity) in a silicon crystal can take the place of one of the tetravalent Si atoms in the crystal structure. Four of its five valence electrons will participate in covalent bonds with its four Si neighbors, and the remaining electron is weakly bound by the resultant positive ion. The Schrödinger equation describing this bound state is

$$\left[-\frac{\hbar^2}{2m^*}\nabla^2 - \frac{e^2}{\epsilon|\mathbf{r}|}\right]\Psi(\mathbf{r}) = E\Psi(\mathbf{r}),$$

where $m^* = 0.2m$ is the effective mass in the conduction band of silicon (m being the free electron mass) and $\epsilon = 11.7$ is the static dielectric constant of silicon.

a) Show that the effective Bohr radius of the bound state is

$$a_d = \frac{\epsilon\hbar^2}{m^*e^2}.$$

Give the numerical value for a_d in Si.

b) Show that the ionization energy of the donor is

$$E_d = \frac{e^4 m^*}{2\epsilon^2\hbar^2}.$$

Give the value of E_d in eV for Si.

c) Consider a crystal of silicon with a density n_d of pentavalent donor impurities per unit volume. If the mean spacing between impurities is much greater than a_d, then each donor electron will be bound to its parent impurity, as discussed above. However, above a critical density n_c, the donor electrons will be delocalized, forming an *impurity band.* Such a material is referred to as an n-type semiconductor, and behaves like a semimetal. Estimate n_c in silicon. Give your answer in units of m^{-3}.