Exercises for Physics 560A

Problem Set 8; Due Friday, November 18

Thermopower of a free-electron metal

Consider a metal specimen subject to a thermal gradient ∇T. The system is an open circuit, so no net electric current can flow. Instead, an electric field $\mathbf{E}$ will be built up to counteract the flow of electrons induced by the thermal gradient:

$$\mathbf{E} = S\nabla T,$$

where the coefficient S is known as the *thermopower*.

The steady-state Boltzmann equation describes the nonequilibrium distribution $f(\mathbf{r}, \mathbf{p})$ of electrons in the specimen:

$$\mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f}{\partial \mathbf{p}} = -\frac{f - f_0}{\tau},$$

where

$$f_0(\mathbf{r}, \mathbf{p}) = \frac{1}{\exp\{\beta(\mathbf{r})[\varepsilon(\mathbf{p}) - \mu(\mathbf{r})]\} + 1}$$

is the Fermi-Dirac distribution in local equilibrium. Consider free electrons, with $\mathbf{p} = m\mathbf{v}$.

a) Assume that $\mathbf{E}$ and ∇T are small, so that $f = f_0 + f_1$, where $f_1 \ll f_0$. Determine f_1 to first order in the temperature gradient and the electric field. For the purposes of this problem, you may set $\nabla\mu = 0$.

b) The electric current density is given by

$$\mathbf{j}_e = -e \int \frac{d^3p}{h^3} \mathbf{v}\, f_1.$$

For a temperature gradient $\partial T/\partial x$ in the x direction, determine the electric field E_x required to stop the flow of electrons ($\mathbf{j}_e = 0$), and hence determine the thermopower S. You may express S as an integral over momentum. Hint: Recall that

$$\int \frac{d^3p}{h^3} p_i \frac{\partial f}{\partial p_j} = -n\delta_{ij}.$$

c) Assume $k_B T \ll \varepsilon_F$. Calculate S to leading order in the temperature. Hint: It may be helpful to change from an integral over momentum to an integral over energy $\rightarrow$ use the *Sommerfeld expansion*.