Exercises for Physics 570A

Problem Set 1; Due 4pm Friday August 31

1) Sakurai, problem 2.38

2) Minimum energy of a harmonic oscillator

The energy of a simple harmonic oscillator is

$$E = \frac{p_x^2}{2m} + \frac{m\omega^2 x^2}{2},$$

where ω is the natural frequency. Use the uncertainty principle

$$\Delta x \Delta p_x \geq \hbar/2$$

to determine a lower bound on the (average) energy $\langle E \rangle$ of the oscillator.

3) Aharonov-Bohm ring

Consider a one-dimensional system in the form of a ring of circumference L threaded by a magnetic flux Φ. The Schrödinger equation for a particle of charge q is

$$E\psi(x) = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx} - \frac{q}{c}A_x(x)\right)^2 \psi(x), \qquad \Phi = \int_0^L A_x(x)dx,$$

where x is a coordinate describing the arc length around the ring, and the wavefunction obeys the boundary condition $\psi(x + L) = \psi(x)$. Note: You may assume that $\mathbf{B} = 0$ along the circumference of the ring.

a) Find the energy eigenvalues and eigenfunctions. Hint: exploit gauge invariance to make the problem easier.

b) Determine the electric current as a function of Φ for a single particle in the ground state.