

Exercises for Physics 570A

Problem Set 7; Due 4pm Friday October 26

1–2) Sakurai 2.26, 2.39

3) Use the WKB approximation to find the allowed energies of the harmonic oscillator.

4) Scattering from a delta-function potential

Consider a particle moving in one dimension with a potential $V(x) = \Lambda\delta(x)$.

a) Calculate the transmission amplitude t and the reflection amplitude r .

b) Calculate the transmission probability $T = |t|^2$ and the reflection probability $R = |r|^2$. Simplify your expressions by introducing the “scattering length” $\ell = \hbar^2/m\Lambda$.

c) Show that the reflection and transmission amplitudes may be written

$$r = \sqrt{R}e^{i\theta}, \quad t = i\sqrt{T}e^{i\theta},$$

and find the function $\theta(k)$. (Hint: \tan^{-1} is a multi-valued function. You must choose the correct branch.)

5) Resonant tunneling through a double barrier potential

a) Show that the transmission probability through a symmetric double barrier is

$$T_{12} = \frac{T^2}{T^2 + 4R \sin^2(kL + \theta)},$$

where T , R , and θ are the transmission probability, reflection probability, and scattering phase shift, respectively, for a single barrier, and L is the free-propagation distance between the two barriers.

b) Next consider the specific potential

$$V(x) = \Lambda\delta(x) + \Lambda\delta(x - L).$$

Using your results from problem 4, plot T_{12} vs. k for the double delta-function barrier, using *Mathematica*, or some other computer graphic utility. Consider the case $L = 10\ell$. Discuss your result; what happens if you vary L and/or ℓ ?