Exercises for Physics 570A

Problem Set 4; due in class September 26

1) Virial theorem

Prove that for a particle moving in a one-dimensional potential $V(x)$, the following equality

$$2\langle T\rangle = \langle x\partial V/\partial x\rangle .$$

holds for stationary states.

2) Scattering from a delta-function potential

Consider a particle moving in one dimension with a potential $V(x) = \Lambda\delta(x)$.

a) Calculate the transmission amplitude t and the reflection amplitude r.

b) Calculate the transmission probability $T = |t|^2$ and the reflection probability $R = |r|^2$. Simplify your expressions by introducing the "scattering length" $\ell = \hbar^2/m\Lambda$.

c) Show that the reflection and transmission amplitudes may be written

$$r = \sqrt{R}\,e^{i\theta}, \quad t = i\sqrt{T}e^{i\theta},$$

and find the function $\theta(k)$. (Hint: $\tan^{-1}$ is a multi-valued function. You must choose the correct branch.)

3) Resonant tunneling through a double barrier potential

a) Show that the transmission probability through a symmetric double barrier is

$$T_{12} = \frac{T^2}{T^2 + 4R\sin^2(kL+\theta)},$$

where T, R, and θ are the transmission probability, reflection probability, and scattering phase shift, respectively, for a single barrier, and L is the free-propagation distance between the two barriers.

b) Next consider the specific potential

$$V(x) = \Lambda\delta(x) + \Lambda\delta(x-L).$$

Using your results from problem 3, plot T_{12} vs. k for the double delta-function barrier, using *Mathematica*, or some other computer graphic utility. Consider the case $L = 10\ell$. Discuss your result; what happens if you vary L and/or ℓ?