Phys 570A HW 4 Solutions

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1) $$[\vec{x}\cdot\vec{p}, \hat{H}] = \frac{1}{2m}[\vec{x}\cdot\vec{p}, \vec{p}^2] + [\vec{x}\cdot\vec{p}, V(\vec{x})]$$

$$= \frac{1}{2m}\left[xp_x + yp_y + zp_z, p_x^2 + p_y^2 + p_z^2\right]$$

$$+ \left[xp_x + yp_y + zp_z, V(x,y,z)\right]$$

$$= \frac{i\hbar}{m}\left(p_x^2 + p_y^2 + p_z^2\right) + \frac{\hbar}{i}\left(x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}\right)$$

$$= i\hbar\left(\frac{\vec{p}^2}{m} - \vec{x}\cdot\nabla V(\vec{x})\right)$$

$$\frac{d}{dt}\langle\vec{x}\cdot\vec{p}\rangle = \frac{1}{i\hbar}\langle[\vec{x}\cdot\vec{p}, \hat{H}]\rangle = \left\langle\frac{\vec{p}^2}{m} - \vec{x}\cdot\nabla V\right\rangle$$

For a stationary state $\frac{d}{dt}\langle\vec{x}\cdot\vec{p}\rangle = 0$,

so $$\left\langle\frac{\vec{p}^2}{m}\right\rangle = \langle\vec{x}\cdot\nabla V\rangle$$

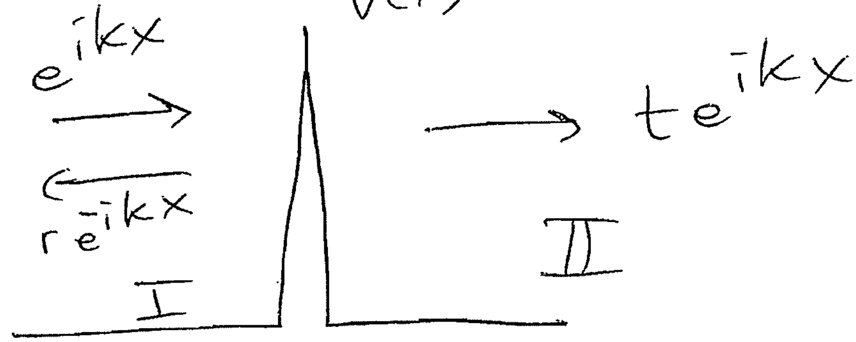
(virial theorem).

Physics 570A HW #4

Solutions

$V(x) = \Lambda\,\delta(x)$

2)



i) $\psi_{I}(0) = \psi_{II}(0) \Rightarrow 1 + r = t$

ii) $\psi'_{II}(0) - \psi'_{I}(0) = \frac{2m\Lambda}{\hbar^2}\psi_{II}(0)$

$$ikt - ik(1-r) = \frac{2m\Lambda}{\hbar^2}\,t$$

$$t + r - 1 = \frac{2m\Lambda}{i\hbar^2 k}\,t = \frac{2t}{ik\ell}$$

use $r = t - 1$

$$\left(2 - \frac{2}{ik\ell}\right)t = 2$$

2

$$t = \frac{1}{1 - \frac{1}{ik\ell}} = \frac{ik\ell}{ik\ell - 1}$$

a)

$$r = t - 1 = \frac{1}{ik\ell - 1}$$

b)

$$T = |t|^2 = \frac{(k\ell)^2}{1 + (k\ell)^2}$$

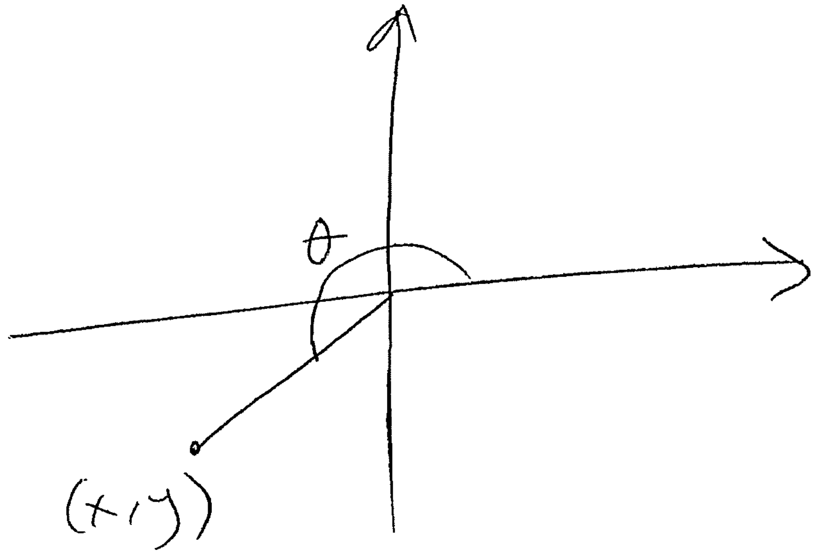
$$R = |r|^2 = \frac{1}{1 + (k\ell)^2}$$

c)

$$r = \sqrt{R}\, e^{i\theta} = \frac{-ik\ell - 1}{1 + (k\ell)^2} = x + iy$$

$$x = -\frac{1}{1 + (k\ell)^2} \qquad y = \frac{-k\ell}{1 + (k\ell)^2}$$

$$\tan\theta = \frac{y}{x}$$ θ in 3rd quadrant



$$\theta(k) = \pi + \tan^{-1}(k\ell)$$

$$t = ik\ell r = ik\ell\sqrt{R}\,e^{i\theta}$$

$$T = (k\ell)^2 R \qquad \sqrt{T} = k\ell\sqrt{R}$$

$$\therefore\ t = i\sqrt{T}\,e^{i\theta}$$

$$S = e^{i\theta}\begin{pmatrix} \sqrt{R} & i\sqrt{T} \\ i\sqrt{T} & \sqrt{R} \end{pmatrix}$$

3) $$T_{12} = \frac{1}{1 + \frac{4R}{T^2}\sin^2(kL + \theta(k))}$$

$$\frac{4R}{T^2} = \frac{4(1+(k\ell)^2)}{(k\ell)^4}$$

$$\theta(k) = \pi + \tan^{-1}(k\ell)$$

$\Rightarrow$ Plot T_{12} vs. $k\ell$

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In[1]:= teta[x_] := -Pi + ArcTan[x]

In[25]:= a := 3 Pi

In[3]:= T[x_] := 1/ (1 + 4 (1+x^2) (Sin[a x + teta[x]])^2 /x^4)

In[27]:= Plot[T[x], {x, 0.1, 1}]
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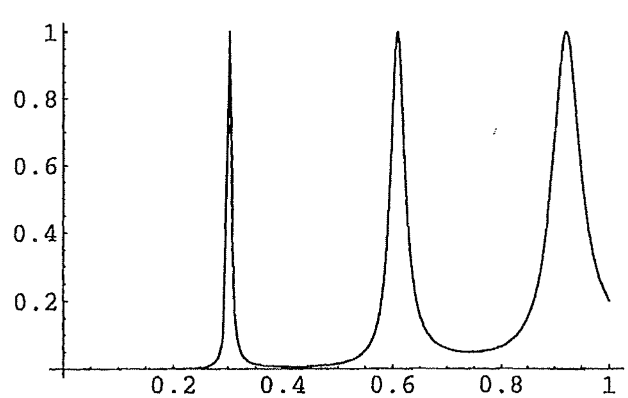

Discussion:

c.f. Lecture 21

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Out[27]= - Graphics -
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