Exercises for Physics 570A

Problem Set 5; due in class October 15

1. (a) Evaluate the classical Poisson bracket

$$\{x, F(p_x)\}.$$

(b) Evaluate the quantum commutator

$$[\hat{x}, \exp(i\hat{p}_x a/\hbar)].$$

(c) Using the result obtained in (b), prove that

$$\exp(i\hat{p}_x a/\hbar)|x'\rangle$$

is an eigenstate of position, where $\hat{x}|x'\rangle = x'|x'\rangle$. What is the corresponding eigenvalue?

2. Show what happens to the expectation values $\langle\vec{x}\rangle$ and $\langle\vec{p}\rangle$ of position and momentum under a translation

$$|\psi\rangle \to \exp(i\vec{p}\cdot\vec{a}/\hbar)|\psi\rangle.$$

3. What is the physical significance of

$$\exp(i\hat{x}Q/\hbar),$$

where Q is some number with the dimensions of momentum?

4. Let $\hat{x}(t)$ be the coordinate operator for a free particle in one dimension in the Heisenberg picture. Evaluate

$$[\hat{x}(t), \hat{x}(0)].$$

5. Consider a free-particle wave packet in one dimension. At $t = 0$, it satisfies the minimum uncertainty relation

$$\Delta x \Delta p_x = \hbar/2 \quad (t = 0).$$

Using the Heisenberg picture, obtain

$$\langle\Delta\hat{x}(t)^2\rangle \text{ and } \langle\Delta\hat{p}_x(t)^2\rangle,$$

where $\Delta\hat{x}(t) \equiv \hat{x}(t) - \langle x(t)\rangle$ and $\Delta\hat{p}_x(t) \equiv \hat{p}_x(t) - \langle p_x(t)\rangle$. Hint: You may use the fact that the initial minimum uncertainty wave packet satisfies

$$\Delta\hat{x}(0)|\psi\rangle = \lambda\Delta\hat{p}_x(0)|\psi\rangle,$$

where λ is pure imaginary (see Shankar, pp. 237-241).