Physics 570A Homework 6

Due in class October 24

Coherent states of the harmonic oscillator

Consider the following initial state of a simple harmonic oscillator:

$$|\Psi(t=0)\rangle = \hat{T}(a)|0\rangle \equiv \exp(-i\hat{p}_x a/\hbar)|0\rangle,$$

where $\hat{T}(a)$ is the translation operator and $|0\rangle$ is the ground state.

a) Show that $|\Psi(t=0)\rangle$ is a *coherent state* $|\lambda\rangle$, that is, an eigenstate of the annihilation operator

$$a|\lambda\rangle = \lambda|\lambda\rangle,$$

and determine the eigenvalue λ. The following identity may be useful: $e^{A+B} = e^A e^B e^{-[A,B]/2}$, for $[A, B]$ a c-number.

b) Show that a general coherent state may be written

$$|\lambda\rangle = e^{-|\lambda|^2/2} e^{\lambda a^\dagger}|0\rangle,$$

where λ is a complex number.

c) Determine $|\Psi(t)\rangle$, and show that (up to a trivial overall phase) it is also a coherent state with time-dependent eigenvalue $\lambda(t)$.

d) Determine $\langle x(t)\rangle$ and $\langle p_x(t)\rangle$.

e) Determine $\Delta x(t)$ and $\Delta p_x(t)$. What is $\Delta x(t)\Delta p_x(t)$?