Physics 570A Homework 7
Due in class October 31

1) Aharonov-Bohm ring

Consider a one-dimensional system in the form of a ring of circumference L threaded by a magnetic flux Φ. The Schrödinger equation for a particle of charge q is

$$E\psi(x) = \frac{1}{2m}\left(\frac{\hbar}{i}\frac{d}{dx} - \frac{q}{c}A_x(x)\right)^2 \psi(x), \qquad \Phi = \int_0^L A_x(x)dx,$$

where x is a coordinate describing the arc length around the ring, and the wavefunction obeys the boundary condition $\psi(x+L) = \psi(x)$. Note: You may assume that $\mathbf{B} = 0$ along the circumference of the ring.

a) Find the energy eigenvalues and eigenfunctions. Hint: exploit gauge invariance to make the problem easier.

b) Determine the electric current as a function of Φ for a single particle in the ground state.

2) Define the velocity operator $\mathbf{v}$ via

$$m\mathbf{v} = \mathbf{p} - q\mathbf{A}/c.$$

Show that the Cartesian components of the velocity operator have the commutation rules

$$[v_x, v_y] = i(e\hbar/m^2c)B_z,$$
$$[v_y, v_z] = i(e\hbar/m^2c)B_x,$$
$$[v_z, v_x] = i(e\hbar/m^2c)B_y.$$

This means that a charged particle in a magnetic field cannot simultaneously have definite values of the velocity components in all three directions.

3) Lowest Landau level

Using the results of problem 2, and the generalized uncertainty principle

$$\Delta A \Delta B \geq \frac{1}{2}|\langle[A, B]\rangle|,$$

show that the kinetic energy of a particle moving in a constant magnetic field is bounded by $E \geq \hbar\Omega/2$, where $\Omega = eB/mc$ is the cyclotron frequency.