Exercises for Physics 570A

Problem Set 8; Due in class Thursday, November 14

1) Evaluate the Feynman path integral

$$\langle x', t'|x, t\rangle = \int_{x=x(t)}^{x'=x(t')} \mathcal{D}[x(t)]e^{i\frac{S[x(t)]}{\hbar}}$$

explicitly for a free particle using the expression given in lec. 16 for fixed N. Hint: Start with N=1, and use induction. See also the discussion in Sankar, Chapter 8.

- 2) Consider the Feynman path integral for a particle moving in one dimension with Hamiltonian $H = \frac{p^2}{2m} + V(x)$.
- a) Show that

$$\frac{\partial}{\partial x'} \langle x', t' | x, t \rangle = \int \mathcal{D}[x(t)] e^{i\frac{S[x(t)]}{\hbar}} \frac{ip_x(t')}{\hbar}.$$

b) Show that

$$\frac{\partial}{\partial t'} \langle x', t' | x, t \rangle = \int \mathcal{D}[x(t)] e^{i\frac{S[x(t)]}{\hbar}} \frac{-iH(t')}{\hbar}.$$

c) Using the results from parts (a) and (b), show that the propagator satisfies Schrödinger's equation.