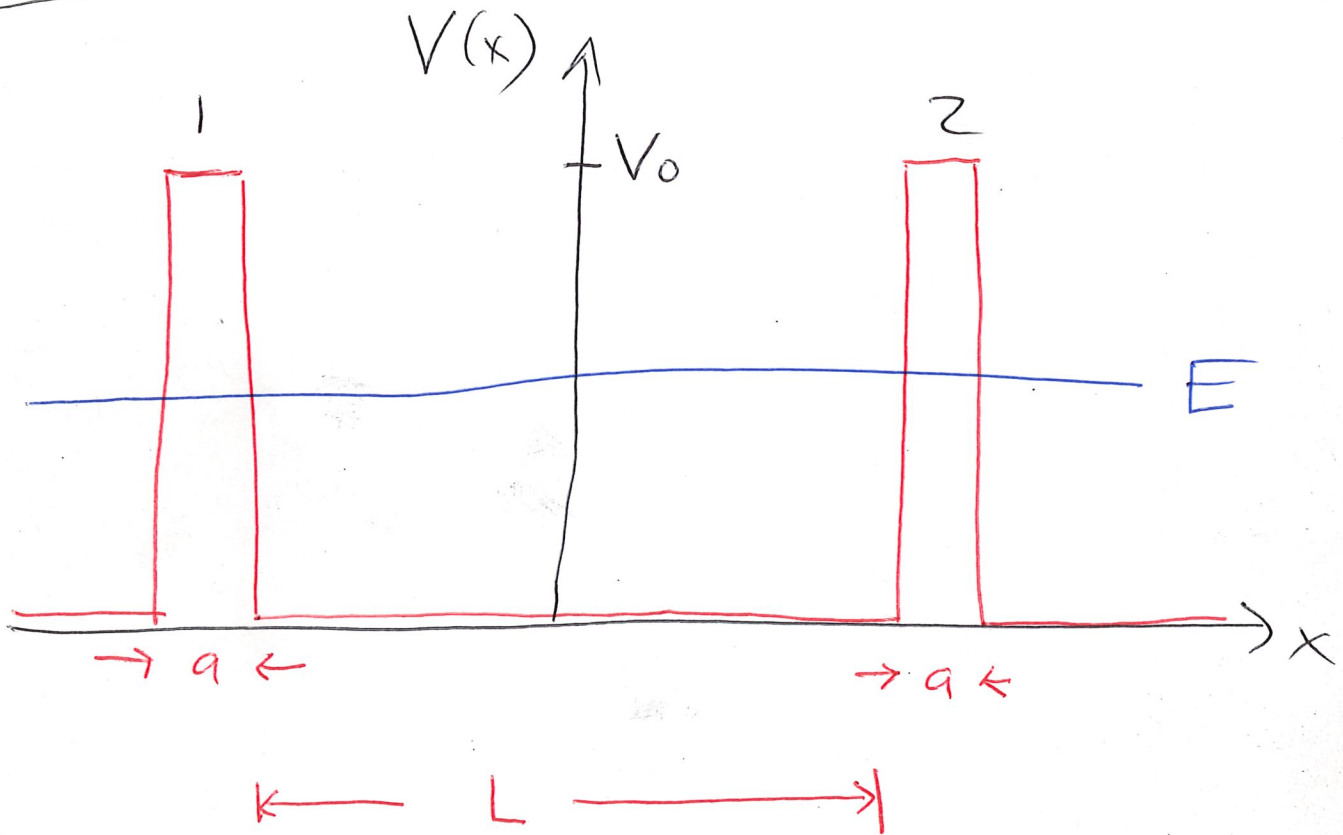


Resonant tunneling through a double barrier



What is the transmission probability? Classically,

$T_{12} = T_1 T_2$ , so the

Probability would be much lower than for a single barrier. Quantum mechanically, we need to consider the interference between the transmitted wave and the multiply-reflected waves. (2

The total transmission amplitude is a geometric series

$$t_{12} = t_1 e^{ikL} t_2 + t_1 e^{ikL} r_2 e^{ikL} r_1 e^{ikL} t_2 + \dots$$

$$= t_1 t_2 e^{ikL} \left( 1 + r_1 r_2 e^{i2kL} + (r_1 r_2 e^{i2kL})^2 + \dots \right)$$

$$= \frac{t_1 t_2 e^{ikL}}{1 - r_1 r_2 e^{i2kL}}$$

For identical barriers,

$$t_1 = t_2 = iT^{1/2} e^{i\theta}$$

$$r_1 = r_2 = R^{1/2} e^{i\theta} \quad (\text{symmetric barriers})$$

$$t_{12} = - \frac{T e^{i(kL+2\theta)}}{1 - R e^{i(2kL+2\theta)}}$$

Total transmission probability

$$T_{12} = |t_{12}|^2 = \frac{T^2}{(1 - R e^{i\alpha})(1 - R e^{-i\alpha})}$$

$$= \frac{T^2}{1 + R^2 - 2R \cos \alpha} = \frac{T^2}{1 + R^2 - 2R \cos(2kL + 2\theta)}$$

$$\cos x = 1 - 2\sin^2 \frac{x}{2}$$

(4)

$$\begin{aligned} T_{12} &= \frac{T^2}{1 - 2R + R^2 + 4R \sin^2(kL + \theta)} \\ &= \frac{T^2}{T^2 + 4R \sin^2(kL + \theta)} \end{aligned}$$

$\Rightarrow$  Resonances ( $T_{12} = 1$ ) when

$kL + \theta = n\pi$ . This was

precisely the condition for

a bound state in the

potential well (c.f. lecture 10).

Q: What is the function(s)

form  $T(E)$  near such a resonance?

$$Le + kL + \theta = n\pi + \varepsilon$$

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$$\sin^2(kL + \theta) \approx \varepsilon^2$$

$$T_{12} \approx \frac{T^2}{T^2 + 4R\varepsilon^2}$$

$$\varepsilon = kL + \theta - n\pi$$

$$k_n L + \theta(k_n) = n\pi$$

$$k = k_n + \Delta k$$

$$\Delta k L + \theta'(k_n) \Delta k \approx \varepsilon$$

$$\varepsilon = \Delta k (L + \theta'(k_n)) = \frac{\Delta E}{\hbar v_n} (L + \theta'(k_n))$$

$$\Delta k = \frac{\Delta E}{\frac{dE}{dk}|_{k_n}} = \frac{\Delta E}{\hbar v_n}$$

$\frac{2(L + \theta'(k_n))}{v_n}$

can be

interpreted as the period of the motion within the well for the  $n$ th standing wave. This is clear

if  $L$  is sufficiently large ( $L \gg \theta'(k_n)$ ). The additional term is known as the

"Wigner delay," and can be thought of as the time the particle dwells within the barrier when

it is reflected.



Q: If we placed a particle in the well with  $k \approx k_n$ , how fast would it leak out?

$$\frac{\Gamma_n}{2} = \frac{\hbar T(E_n) v_n}{2(L + \theta'(k_n))} = \frac{\hbar}{\tau_1}$$

$\tau_1$  = escape time through a single barrier.

In terms of  $\Gamma_n$ , one

has

$$\circ T_{12}(E) \approx \frac{(\Gamma_n/2)^2}{R_n(E - \bar{E}_n)^2 + (\Gamma_n/2)^2} \quad \left\{ 8 \right.$$

This form of resonance is known as Breit-Wigner resonance. The full

width at half maximum is

$$\Delta E = \sqrt{R_n} \Gamma_n$$

$\approx \Gamma_n$  for nearly opaque barriers.

Thus the width of the



resonance is related to the  
escape time of the quasi-bound  
state

$$\Delta E \tau = \Gamma_n \frac{\hbar}{\Gamma_n} = \hbar$$

Via the energy-time  
uncertainty relation!

$$\text{Here } \frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} = \frac{2}{\tau_1}$$

Resonances such as this  
arise in many fields of  
physics!