Physics 570A Lecture 2

1) The ground state of the hydrogen atom and the uncertainty principle

Bohr:
$$E_n = -\frac{me^4}{2\hbar^2}\frac{1}{n^2}$$
$$n = 1, 2, 3, \ldots$$

"Why" is no energy lower than

$$E_1 = -\frac{me^4}{2\hbar^2}$$ possible?

$$E = \frac{p^2}{2m} - \frac{e^2}{r}, \quad \Delta p \, \Delta r \gtrsim \hbar$$

Energy is lowered by decreasing r and/or p. Best we can do is

$$E \sim \frac{\Delta p^2}{2m} - \frac{e^2}{\Delta r}$$

$$= \frac{\Delta p^2}{2m} - \frac{e^2 \Delta p}{\hbar} \qquad \left(\text{using } \Delta r = \frac{\hbar}{\Delta p}\right)$$

$$\min E \Rightarrow \quad 0 = \frac{\partial E}{\partial \Delta p} = \frac{\Delta p}{m} - \frac{e^2}{\hbar}$$

$$\Delta p = \frac{me^2}{\hbar} \qquad \Delta r = \frac{\hbar}{\Delta p} = \frac{\hbar^2}{me^2}$$

(Bohr radius!)

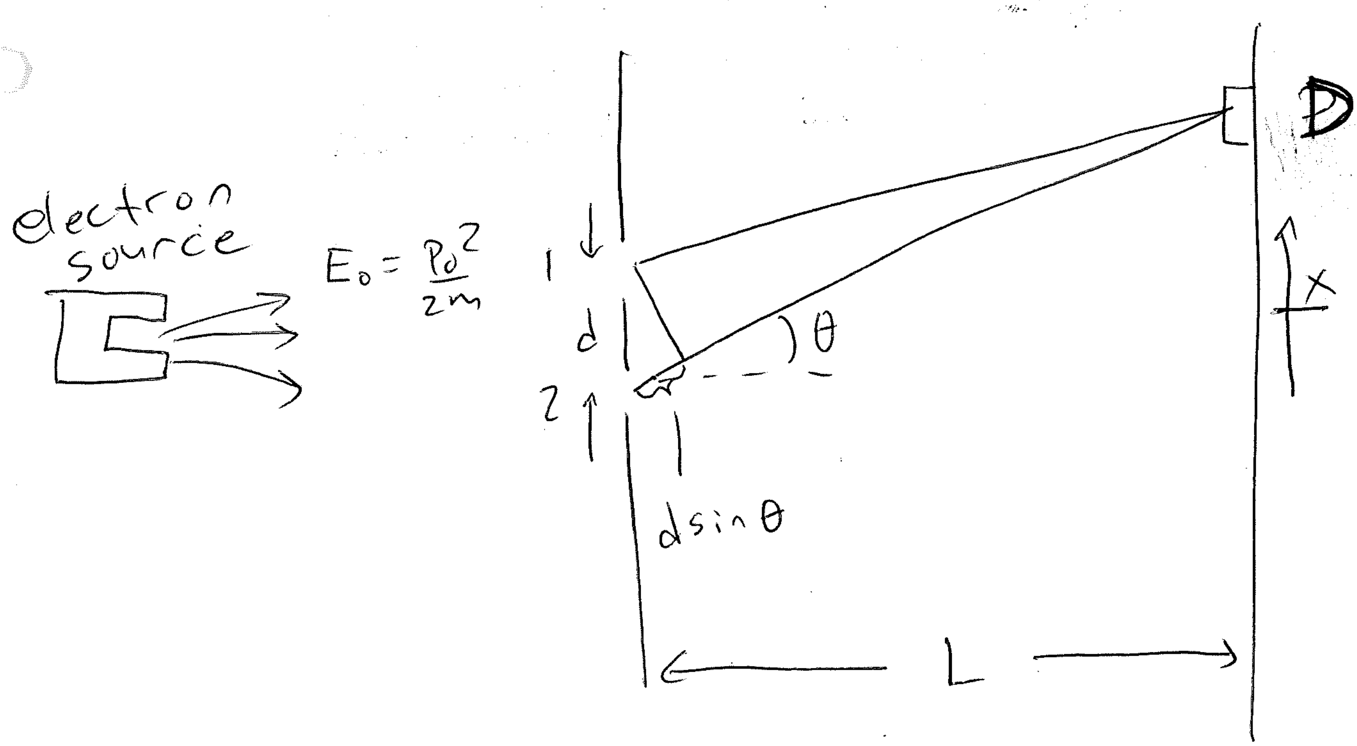
$$E = \frac{1}{2m}\left(\frac{me^2}{\hbar}\right)^2 - \frac{e^2 me^2}{\hbar^2} = -\frac{1}{2}\frac{me^4}{\hbar^2}$$

$$= -13.6\,\text{eV}$$

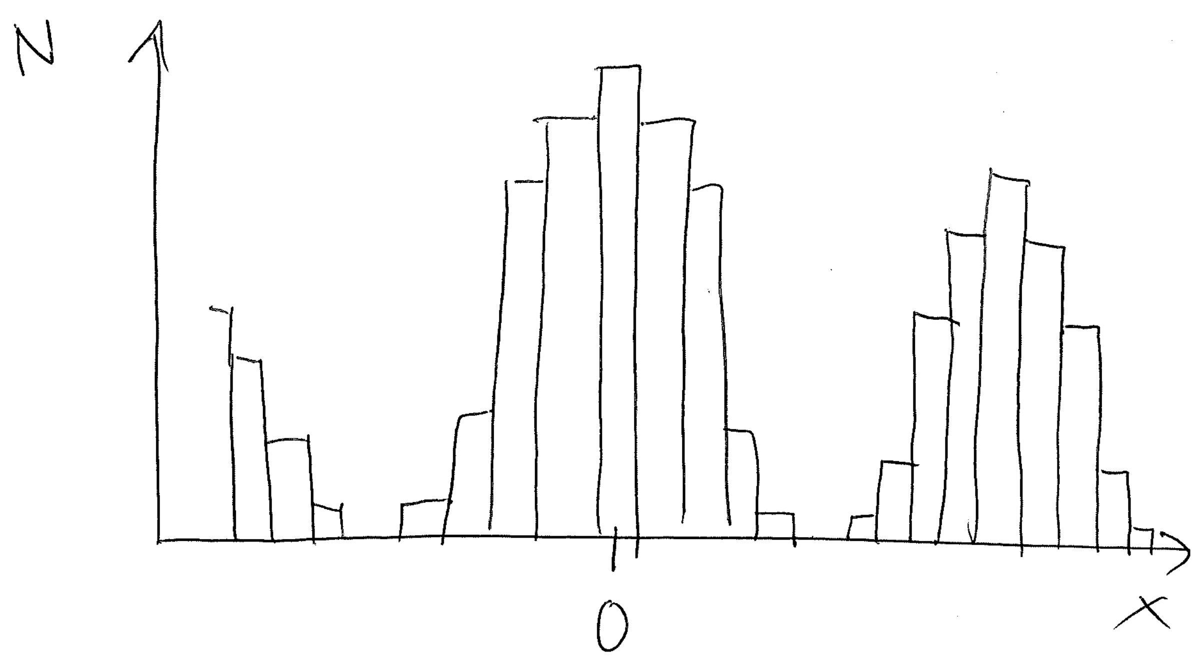
Thus the first Bohr orbital is the lowest possible energy of the hydrogen atom, consistent with the uncertainty principle!

2) Wave-particle duality:

The double-slit experiment and the uncertainty principle



An electron gun emits electrons of energy E_0 toward a screen with two slits, with separation d. What is observed on a screen a distance $L \gg d$ away? Electrons are observed to hit the screen one by one at particular points x, corresponding to angles $\theta \simeq x/L$. After a long time, if we count all the electrons which have hit the screen, we find



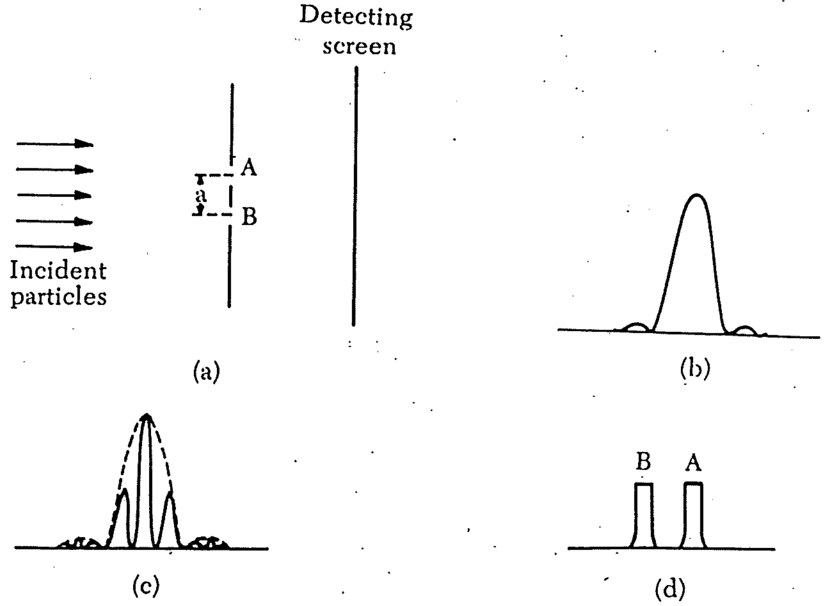
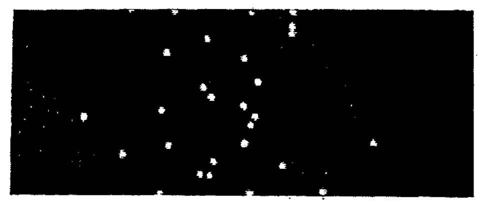
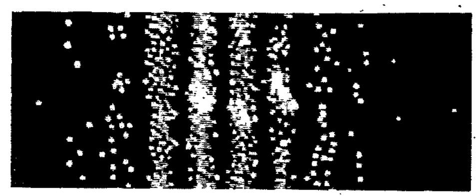


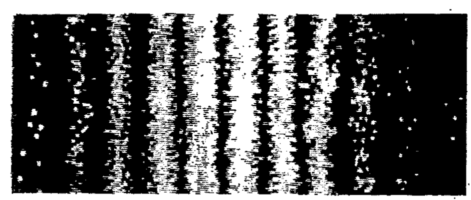
Figure 4-9 (a) Double slit experiment with particles. (b) Distribution of particles recorded on the screen due to diffraction from either slit A or B. (c) Distribution of particles recorded on the screen due to diffraction with both slits A and B open. (d) Hypothetical distribution of particles recorded on the screen if wave effects are neglected.



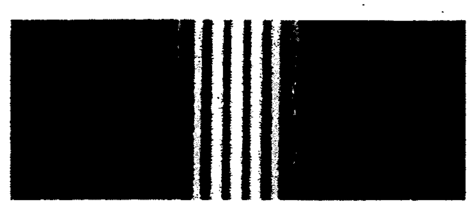
(a) After 28 electrons



(b) After 1000 electrons



(c) After 10,000 electrons



(d) Two slit electron pattern

Figure 4-10 (a), (b), and (c). Computer-simulated growth of a two-slit interference pattern for electrons. (d) An actual photograph of a two-slit pattern produced by electrons. (Parts (a), (b), and (c) from E. R. Huggins, *Physics I*, W. A. Benjamin, Inc., New York, 1968. Part (d) is from C. Jönsson, *Zeitschrift für Physik*, **161**, 454 (1961). Used with permission.)

The maximum numbers of counts occur for constructive interference of the de Broglie waves:

$n = 0, \pm 1, \pm 2, \ldots$

$$d \sin\theta = n \lambda_e, \qquad \lambda_e = \frac{h}{p_0}$$

The minima correspond to destructive interference:

$$d \sin\theta = \left(n + \frac{1}{2}\right) \lambda_e, \qquad n = 0, \pm 1, \pm 2, \ldots$$

Nonetheless, each electron hits the screen in one and only one place. How can we reconcile the interference pattern with the very discrete nature of the electron?

Proposition A: Each electron passes through <u>either</u> slit 1 <u>or</u> slit 2.

The particle nature of the electron would seem to imply the validity of Prop. A. Yet the interference pattern accumulates even if we slow down the electron source so that at most one electron impinges on the two slits at any given time. Interference would seem to require the electron to pass through both slits.

To resolve this apparent paradox, we should test prop. A directly, e.g., by shining some light on the slits, to <u>see</u> which slit the electron passes through. In order to resolve which slit the electron went through, we need to use light of wavelength $\lambda_\gamma < d$. If the light source is bright enough, then each electron scatters one or more photons in passing through the slits, and we find that prop. A is indeed true. However, each photon has momentum

$p_\gamma = \frac{h}{\lambda_\gamma}$. The momentum

p_0 of the electron is thus changed by an amount

$\Delta p \sim \frac{h}{\lambda_\gamma}$, which changes the "trajectory" by an amount $\Delta\theta \sim \frac{\Delta p}{p_0} = \frac{\lambda_e}{\lambda_\gamma}$.

The angular separation of

neighboring maxima and minima of the interference pattern is

$$d\left|\sin\theta_{max} - \sin\theta_{min}\right| = \frac{\lambda_e}{2}$$

$$\left|\theta_{max} - \theta_{min}\right| \sim \frac{\lambda_e}{2d}.$$

If $\Delta\theta > \frac{\lambda_e}{2d}$, then the interference pattern will be smeared out, i.e., if

$$\Delta\theta \sim \frac{\lambda_e}{\lambda_\gamma} > \frac{\lambda_e}{2d} \Rightarrow \lambda_\gamma < 2d.$$

But it is necessary to use light with $\lambda_\gamma < d$ to resolve which slit the

electron passed through!
Thus, if we check prop. A, the interference pattern is destroyed. The seemingly contradictory wave-like and particle-like aspects of the electron cannot be brought into conflict, because observation of one aspect precludes observation of the other aspect. This was shown for the specific example

of light-scattering, but Heisenberg postulated that this is a fundamental feature of the quantum world: It is impossible to design any apparatus capable of simultaneously determining the position and momentum of an object with a joint precision violating the inequality

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}.$$