

The Aharonov-Bohm effect

In addition to the Meissner effect, another interesting consequence of the way the vector potential enters the Schrödinger equation is the Aharonov-Bohm effect, which describes the QM effect of the vector potential, even when $\vec{B} = 0$.

See Sakurai

2.7.

Modified
2-slit exp.

Electron
source



$$E = \frac{\hbar^2 k^2}{2m}$$



$$\vec{B} = 0$$



$$\vec{B} = 0$$



\vec{R}

x

screen

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Magnetic flux Φ due to a solenoid
within central barrier. $\vec{B} = 0$
everywhere the electrons can
propagate.

Time-indep. Sch. eq.

$$E \psi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \frac{q}{c} \vec{A} \right)^2 \psi \quad (V=0)$$

Outside barrier, $\vec{B} = 0$, so

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$$\vec{A} = \nabla f(\vec{r}, t) = \frac{\hbar c}{\phi} \nabla \theta$$

$$E \psi = \frac{1}{2m} \left(\frac{\hbar}{i} \nabla - \hbar \nabla \theta \right)^2 \psi$$

$$\text{Let } \psi = e^{i\theta(\vec{r})} \psi'$$

ψ' satisfies free Sch. eq.

$$E \psi' = -\frac{\hbar^2}{2m} \nabla^2 \psi'$$

Interference pattern

$$P(\vec{r}) = |\psi_1(\vec{r}) + \psi_2(\vec{r})|^2$$

$$= |\psi_1'(\vec{r}) e^{i\theta_1} + \psi_2'(\vec{r}) e^{i\theta_2}|^2$$

$$\psi_1'(\vec{r}) = \sqrt{P_1} e^{ikL_1}, \quad \psi_2'(\vec{r}) = \sqrt{P_2} e^{ikL_2}$$

$$f(\vec{R}) = f_1 + f_2 + 2\sqrt{f_1 f_2} \cos(k\Delta L + \theta_2 - \theta_1)$$

$\Delta L = L_2 - L_1$, = difference in path lengths

$$\theta_1 = \int_1 \frac{q}{\hbar c} \vec{A} \cdot d\vec{l}, \quad \theta_2 = \int_2 \frac{q}{\hbar c} \vec{A} \cdot d\vec{l}$$

$$\theta_2 - \theta_1 = \oint \frac{q}{\hbar c} \vec{A} \cdot d\vec{l}$$

$$= \frac{q}{\hbar c} \Phi = \frac{2\pi \Phi}{\Phi_0}$$

$$\Phi_0 = \frac{\hbar c}{q} = \text{"flux quantum"}$$

Interference pattern is shifted by magnetic flux in solenoid, even though

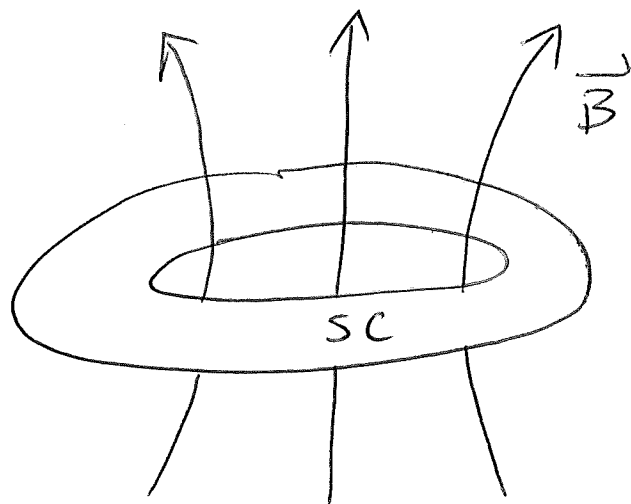
$\vec{B} = 0$ everywhere particle can propagate!
 Shift is periodic in Φ with period Φ_0 .

Flux quantization

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One consequence of the AB effect is the quantization of the magnetic flux piercing a SC ring.

The Meissner effect implies that $\vec{B} = 0$ and $\vec{J}_e = 0$ in the interior of the superconductor.



$$0 = \vec{J}_e = \frac{n_s q}{m} (\hbar \nabla \theta - \frac{q}{c} \vec{A})$$

$$\Rightarrow \nabla \theta = \frac{q}{\hbar c} \vec{A} \quad (\psi_s(\vec{r}) = \sqrt{n_s(\vec{r})} e^{i\theta})$$

The wavefunction of the SC is single-valued, which means $\theta(\vec{r})$ must return to its original value on traversing the ring, or change by

an integer multiple of 2π :

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$$\oint \nabla\theta \cdot d\vec{l} = 2\pi s, \quad s \in \mathbb{Z}$$

||

$$\oint \frac{q}{\hbar c} \vec{A} \cdot d\vec{l} = \frac{q}{\hbar c} \Phi = 2\pi s$$

$$\Phi = \frac{\hbar c}{q} s = s \Phi_0,$$

$$\Phi_0 = \frac{\hbar c}{2e}$$

superconducting
flux quantum

The superconducting surface currents adjust to fix the total magnetic flux through the ring to an integer times Φ_0 . Such persistent currents have been observed to flow, undiminished, for several years.

Persistent Current in a Mesoscopic Au Ring

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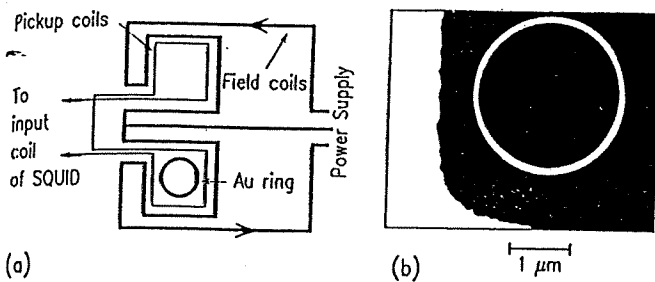
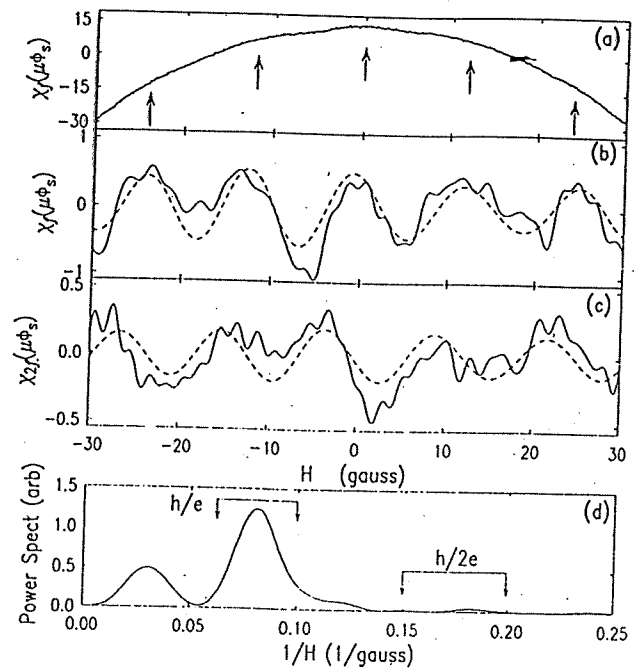


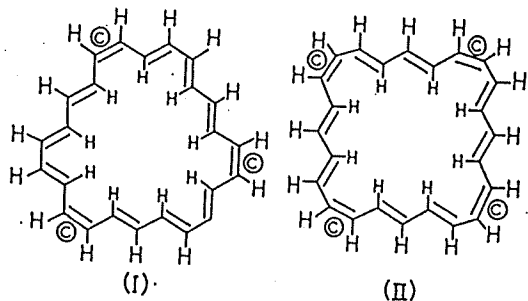
FIG. 1. (a) Schematic diagram of the pickup coil chip, illustrating the counterwound Nb pickup coils and the on-chip magnetic-field coils. (b) Micrograph of the 2.4- μm -diam Au ring in one corner of the 9- μm -inner-diam Nb pickup coil. The loop linewidth is 90 nm and the thickness is 60 nm.



Chandrasekhar et al. 1991

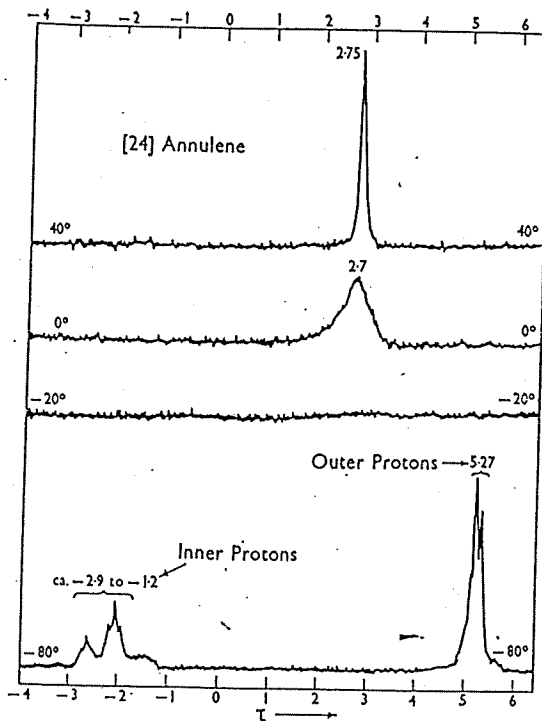
Persistent Current in a Microscopic Ring

to allow a distinction to be made between structure (I) (inner:outer proton ratio, 37.5:62.5) and structure (II) (ratio, 33.3:66.7).



The room temperature n.m.r. spectrum of [24]annulene evidently again represents an average, due to rotation of the carbon-carbon bonds (conformational isomerism). In addition, movement of the π -bonds (valence isomerism) may be involved, as has been postulated for cyclo-octatetraene⁹ and [16]annulene.⁴

It is remarkable that in the low-temperature n.m.r. spectra of the $4n$ π -electron systems, [16]annulene and [24]annulene, the inner protons appear at low field and the outer protons at high field. This is a reversal of the behaviour of the $(4n + 2)$ systems, [14]annulene and [18]annulene. A similar reversal between $4n$ and $(4n + 2)$ systems in the dehydro-annulene series has already been observed,¹⁰ and a theoretical explanation based on quantum-mechanical considerations has been advanced.¹¹



FIGURE

N.m.r. spectrum of [24]annulene, in perdeuteriotetrahydrofuran solution (100 Mc./sec., tetramethylsilane used as internal standard).

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Calder & Sondheimer 1966