Physics 570A Midterm 1 Practice Problems
8.5"x5.5" crib sheet (one side) and scientific calculator allowed.

1) 2-level system

Consider a quantum system with Hamiltonian, position operator, and state vector

$$\hat{H} = \begin{pmatrix} \varepsilon & \delta \\ \delta & \varepsilon \end{pmatrix}, \quad \hat{x} = \begin{pmatrix} a/2 & 0 \\ 0 & -a/2 \end{pmatrix}, \quad \text{and} \quad \psi = \frac{1}{\sqrt{|\alpha|^2 + |\beta|^2}} \begin{pmatrix} \alpha \\ \beta \end{pmatrix},$$

respectively, where α and β are complex numbers and the other parameters are real.

a) If the position x is measured, what are the possible outcomes, and with what probabilities do they occur?

b) If the energy E is measured, what are the possible outcomes, and with what probabilities do they occur?

c) Derive a formal lower bound on the uncertainty product $\Delta x \Delta E$. For what state(s) is this lower bound maximized? What is the maximum value?

2) 1D scattering

Consider a particle of energy E incident from the left on the potential barrier

$$V(x) = \lambda \delta(x) + V_0 \theta(x).$$

a) Determine the transmission and reflection probabilities for the case $E > V_0$.

b) Determine the reflection amplitude (and phase shift) for the case $E < V_0$.