Physics 570A Midterm 2 Practice Problems

8.5"x11" crib sheet (one side) and scientific calculator allowed.

1) Driven harmonic oscillator

Consider a driven harmonic oscillator with Hamiltonian

$$H(t) = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2} - F(t)x,$$

where $F(t) = Ct$ and $[x, p] = i\hbar$.

a) Derive the Heisenberg equations of motion for $x(t)$ and $p(t)$.

b) Solve the Heisenberg equations of motion, and show that the solution can be written

$$x(t) = x(0)\cos\omega t + \frac{1}{m\omega}\left(p(0) - \frac{C}{m\omega^2}\right)\sin\omega t + \frac{Ct}{m\omega^2},$$

$$p(t) = -m\omega x(0)\sin\omega t + \left(p(0) - \frac{C}{m\omega^2}\right)\cos\omega t + \frac{C}{m\omega^2}.$$

c) Discuss your solution qualitatively, and give an interpretation of the resulting motion.

2) Aharonov-Bohm effect

Consider a two-slit experiment with electrons, where a magnetic flux Φ is encapsulated in the impenetrable barrier between the two narrow slits, whose separation is d. Assume a monochromatic source of electron waves of energy $E = \hbar^2 k^2/2m$ illuminating the slits.

The intensity pattern is observed on a screen parallel to the plane of the two slits, a large distance $L \gg d$ away from the slits. At what angles are bright fringes observed? At what angles are dark fringes observed? How do these angles depend on Φ?