Physics 570A Final Exam Practice Problems

8.5"x11" crib sheet (two sides) and scientific calculator allowed.

1) Aharonov-Bohm effect

Consider a two-slit experiment with electrons, where a magnetic flux Φ is encapsulated in the impenetrable barrier between the two narrow slits, whose separation is d. Assume a monochromatic source of electron waves of energy $E = \hbar^2 k^2/2m$ illuminating the slits.

The intensity pattern is observed on a screen parallel to the plane of the two slits, a large distance $L \gg d$ away from the slits. At what angles are bright fringes observed? At what angles are dark fringes observed? How do these angles depend on Φ?

2) 1D Scattering

a) Calculate the transmission probability for a particle of energy $E = V_0/99$ incident from the left on the negative potential step

$$V(x) = -V_0\,\theta(x), \quad \text{where} \quad \theta(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

and $V_0 > 0$. *Note: You must derive your result by solving Schrödinger's equation.*

b) Now suppose the potential step is inverted $V_0 \to -V_0$. Determine the transmission probability. *Again, you must justify your result with a detailed calculation.*

3) Sequential measurements

An operator $\hat{A}$, representing observable a, has two normalized eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable b, has two normalized eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1 = \sqrt{\frac{1}{2}}(\phi_1 + \phi_2), \qquad \psi_2 = \sqrt{\frac{1}{2}}(\phi_1 - \phi_2).$$

a) Observable a is measured, and the value a_1 is obtained. What is the state of the system (immediately) after the measurement?

b) If b is now measured, what are the possible results, and with what probabilities do they occur?

c) Right after the measurement of b, a is measured again. What is the probability of getting a_1 again?

4) Coherent states

Consider a simple harmonic oscillator:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2\hat{x}^2}{2}.$$

a) Construct a normalized state vector $|\lambda\rangle$ which is an eigenfunction of the annihilation operator,

$$\hat{a}|\lambda\rangle = \lambda|\lambda\rangle,$$

where

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\hat{x} - \frac{i\hat{p}}{\sqrt{2m\hbar\omega}}.$$

Such a state is known as a *coherent state.*

b) Is there a corresponding ket that is an eigenstate of the creation operator $\hat{a}^\dagger$? If so, construct it; if not, prove that it doesn't exist.

5) Spin-1/2: Measurement of S_z or S_y

Consider a spin-1/2 particle in the general spin state

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix} \equiv a\psi_\uparrow + b\psi_\downarrow,$$

where a and b are complex numbers, and $\psi_\uparrow$ and $\psi_\downarrow$ are eigenstates of S_z.

a) If a measurement of the z-component of the particle's spin, S_z, is performed, what are the possible outcomes, and with what probabilities do they occur? What is the expectation value $\langle S_z\rangle$?

b) If, instead, a measurement of the y-component of the particle's spin, S_y, is performed, what are the possible outcomes, and with what probabilities do they occur? What is the expectation value $\langle S_y\rangle$?

6) Addition of angular momenta: $1 + 1/2$

Consider the total angular momentum $\vec{J} = \vec{L} + \vec{S}$ of a spin-1/2 particle with wavefunction

$$|\Psi\rangle = |\ell = 1, m_\ell = 1\rangle|\downarrow\rangle.$$

a) If the z-component J_z of the total angular momentum is measured, what are the possible outcomes, and with what probabilities do they occur?

b) If the total angular momentum squared $\vec{J}^2$ is measured, what are the possible outcomes, and with what probabilities do they occur?

7) Spin interferometer

Two electrons are prepared with initial wavefunction

$$|\Psi_i\rangle = |S=1, m_s=0\rangle = \sqrt{1/2}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle).$$

The electrons are then separated and passed through an interferometer such that electron 1 acquires a phase $e^{i\theta}$ if it has spin-up and $e^{-i\theta}$ if it has spin-down, while the phase of electron 2 is unchanged. The total wavefunction of the system after the electrons have traversed the interferometer is

$$|\Psi_f\rangle = \sqrt{1/2}\left(e^{i\theta}|\uparrow\downarrow\rangle + e^{-i\theta}|\downarrow\uparrow\rangle\right).$$

If the total angular momentum (squared) of the system $\vec{S}^2$ is then measured, what are the possible outcomes? With what probabilities do they occur?