Spring break review problems for PHYS 262H

Ch. 3: 39, 40
Show that the reduced mass of a 2-body system is less than either of the constituent masses.
Calculate the speed of an electron in the n-th state of the hydrogen atom in terms of $\alpha, e, m_p, m_e$, and $n$. Remember to treat $m_p$ as finite. If you can correctly work through this problem, your understanding of the Bohr atom is pretty complete.

Ch. 5: 10 (for visible light, also compare the kinetic energy of the electrons to the energy of the photons), 18, 31

Ch. 7: 4, 5, 11, 12, 27, 35
Consider the quantum state $\phi$ with wavefunction

$$\phi(x) = \sqrt{\frac{2}{L}} \left( \frac{3}{5} \sin \frac{\pi x}{L} + \frac{4}{5} \sin \frac{3\pi x}{L} \right)$$

confined to the region $0 \leq x \leq L$.

a) Show that $\phi$ is normalized.

b) Is $\phi$ an energy eigenstate (a solution of Schroedinger’s equation)?

c) Calculate $\langle H \rangle$ (the expectation value of the energy) of $\phi$. Recall that

$$\hat{H} = \frac{\hat{p}^2}{2m} + V(x),$$

but here $V = 0$ in the region of interest.

Hint:

$$\int_0^L \sin \frac{n\pi x}{L} \sin \frac{m\pi x}{L} = 0$$

if $m \neq n$ and $= \frac{2}{L}$ if $m = n$

Eigenfunction expansion: Suppose that we can write

$$\phi = a_1 \psi_1 + a_2 \psi_2 + ...$$

where the $a_i$’s are numbers (possibly complex), and the $\psi_i$’s are orthonormal (orthogonal and normalized) eigenfunctions of the operator $\hat{O}$ with eigenvalues $\lambda_i$. Then the normalization condition on $\phi$ is

$$|a_1|^2 + |a_2|^2 + ... = 1$$

and the expectation value $\langle O \rangle$ of $\phi$ is

$$\langle O \rangle = a_1^2 \lambda_1 + a_2^2 \lambda_2 + ...$$

d) Reevaluate the answers to a) and c) using the above equations. Which way is easier?