Effective Field Theories of Light Nuclei

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Background by S. Hossenfelder

Outline

- Effective Field Theories
- Shallow Bound States:
  - $NN \rightarrow NN$ and $s_1\,(d)$, s_0
  - $Nd \rightarrow Nd$ and $s_{1/2}\,(t)$
- Narrow Resonances:
  - $N\alpha \rightarrow N\alpha$ and $p_{3/2}$
- Outlook

❧ Wanted ☙
Dead ♦ or ♦ Alive

QCD EXPLANATION OF NUCLEAR PHYSICS

Reward

understanding of gross features:

Why is $B/A \sim 10\,\text{MeV} << M_{QCD} \sim 1\,\text{GeV}$?

How large are few-nucleon forces?

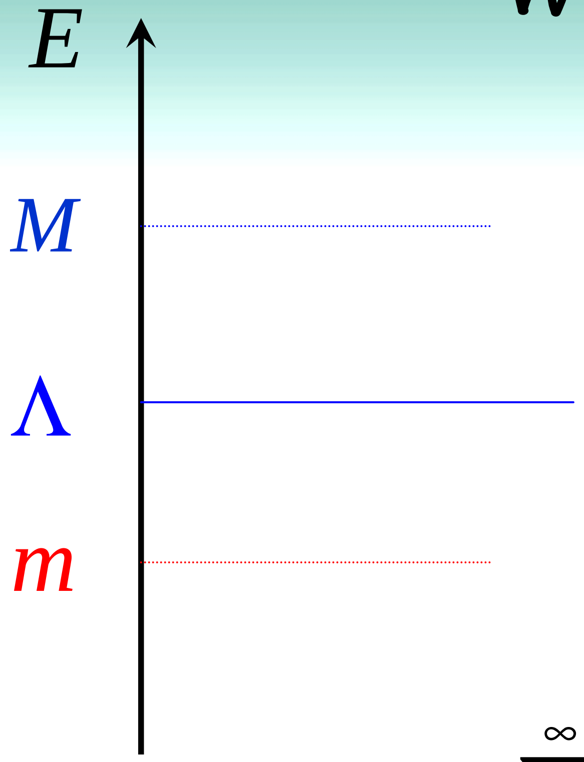
Why is isospin a good symmetry?

…

Beware

coupling constants not small

What is Effective?



$$Z = \boxed{\int D\phi_H \int D\phi_L \exp\left(i\int d^4x\, L_{und}(\phi_H,\phi_L)\right)}$$

$$\times \int D\varphi\, \delta\left(\varphi - f_\Lambda(\phi_L)\right)$$

$$= \int D\varphi\, \boxed{\exp\left(i\int d^4x\, L_{EFT}(\varphi)\right)}$$

$$L_{EFT} = \sum_{d=0}^{\infty} \sum_{i(d,n)} c_i(M,\Lambda)\, O_i\left((\partial\, , m)^d\, \varphi^n\right)$$

renormalization-group invariance $\frac{\partial Z}{\partial \Lambda} = 0$

local

underlying symmetries

$$\begin{cases} T = T^{(\infty)}(Q) \sim \underbrace{N(M)}_{\text{normalization}} \displaystyle\sum_{\nu=\nu_{\min}}^{\infty} \sum_{i} \tilde{c}_{\nu,i}(\Lambda) \left[\frac{Q}{M}\right]^{\nu} \underbrace{F_{\nu,i}\left(\frac{Q}{m};\frac{\Lambda}{m}\right)}_{\text{non-analytic, from loops}} \\ \dfrac{\partial T}{\partial \Lambda} = 0 \end{cases}$$

$$\nu = \nu(d, n, \ldots)$$ “power counting”

↳ e.g. # loops *L*

For $Q \sim m$, truncate consistently with RG invariance

so as to allow systematic improvement (perturbation theory):

$$T = T^{(\nu_{\max})} + O\left(\frac{Q}{M}\right)^{\nu_{\max}+1} \qquad \frac{\partial T^{(\nu_{\max})}}{\partial \Lambda} = O\left(\frac{Q}{\Lambda}\right)^{\nu_{\max}+1}$$

Nuclear physics scales

"His scales are His pride", Book of Job

$\ln Q$

perturbative QCD

~1 GeV $M_{QCD} \sim m_N,\, m_\rho,\, 4\pi f_\pi,\, \ldots$ sum ?

hadronic th with chiral symm

~100 MeV $M_{nuc} \sim f_\pi,\, 1/r_{NN},\, m_\pi,\, \ldots$ sum Q/M_{nuc}

~30 MeV $\aleph \sim 1/a_{NN}$ sum $Q/\aleph$

no small coupling → expansion in Q

Nuclear EFTs

$Q \sim m_\pi << M_{QCD}$

pionful EFT

- degrees of freedom: nucleons, pions (+ deltas, roper?, …)

$$m_\Delta - m_N \sim 2m_\pi\,,\; m_{N'} - m_N \sim 3.5m_\pi$$

- symmetries: Lorentz, $\not{P}$, $\not{T}$, $\cancel{\text{chiral}}$

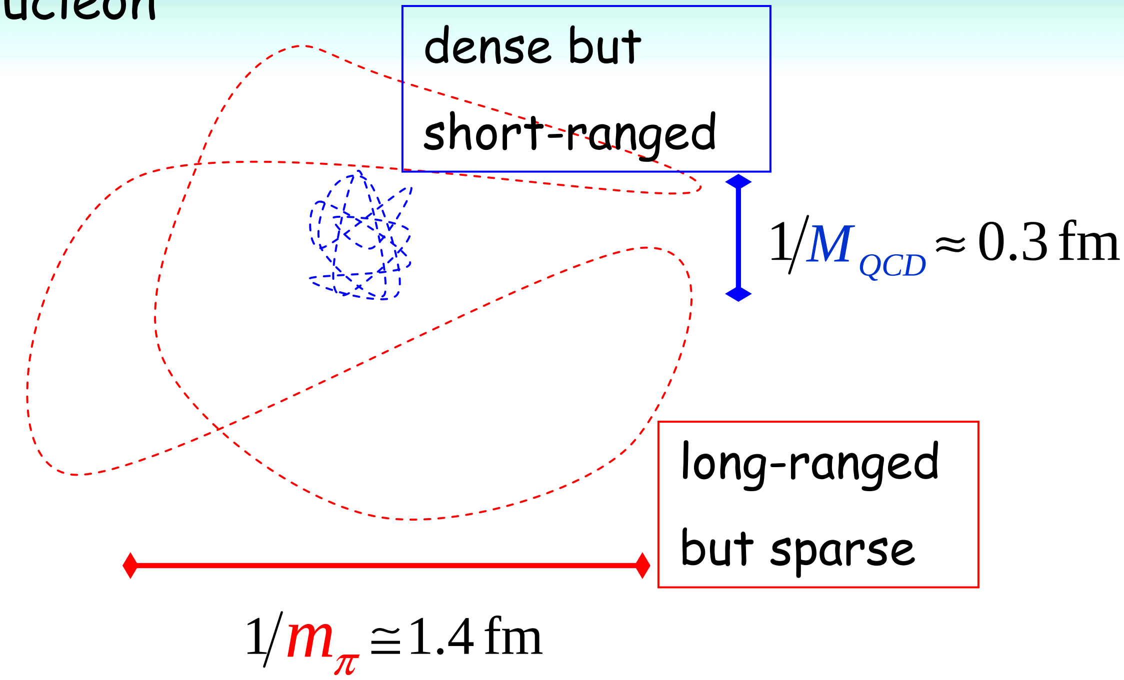
- expansion in:

$$\frac{Q}{M_{QCD}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\rho, \cdots & \text{multipole} \\ Q/4\pi f_\pi & \text{pion loop} \end{cases}$$

A= 0, 1: chiral perturbation theory

Weinberg '79
Gasser + Leutwyler '84
…
Gasser, Sainio + Svarc '87
Jenkins + Manohar '91
…

nucleon



dense but short-ranged

$1/M_{QCD} \approx 0.3\,\mathrm{fm}$

long-ranged but sparse

$1/m_\pi \cong 1.4\,\mathrm{fm}$

Weinberg '90, '92
Ordonez + v.K. '92
…
Kaplan, Savage + Wise '98
…
Beane, Bedaque, Savage + v.K. '02
…
Entem + Machleidt '03
Epelbaum, Glockle + Meissner '04
…

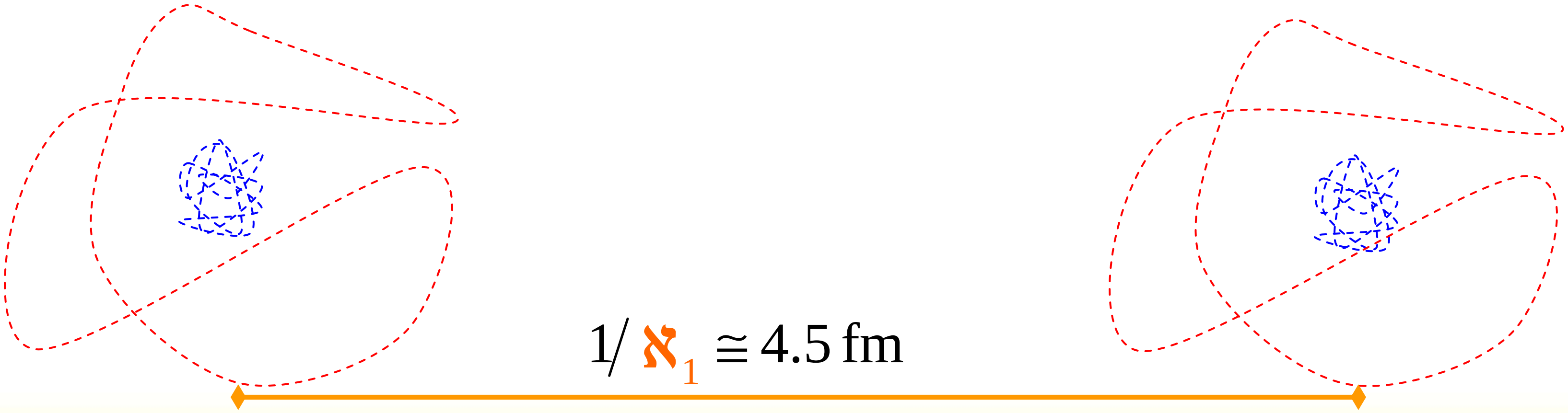
A $\geq$ 2: resummed chiral perturbation theory

- overkill at lower energies!

e.g. NN $\quad s_1$ channel: $\qquad\qquad$ s_0 channel:

(real) bound state = deuteron $\qquad\qquad$ (virtual) bound state

$$\aleph_1 \sim \sqrt{m_N B_d} \cong 45\,\text{MeV} < m_\pi \qquad\qquad \aleph_0 \sim \sqrt{m_N B_{d^*}} \cong 8\,\text{MeV} << m_\pi$$



multipole expansion of meson cloud:

contact interactions among local nucleon fields

$$Q \sim \aleph \ll M_{nuc}$$

pionless EFT

- degrees of freedom: nucleons
- symmetries: Lorentz, $\cancel{P}$, $\cancel{T}$
- expansion in:

$$\frac{Q}{M_{nuc}} = \begin{cases} Q/m_N & \text{non-relativistic} \\ Q/m_\pi, \cdots & \text{multipole} \end{cases}$$

$$L_{EFT} = N^+ \left(i\partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0 N^+ N N^+ N$$

$$+ N^+ \frac{\nabla^4}{8m_N^3} N + C_2 N^+ N N^+ \nabla^2 N$$

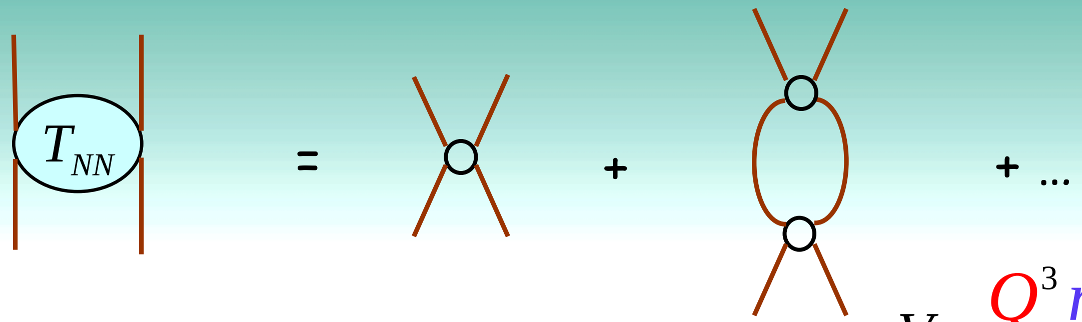
$$+ C_2' N^+ \vec{\nabla} N \cdot N^+ \vec{\nabla} N + \ldots$$

(omitting spin, isospin)

v.K. '97 '99

Kaplan, Savage + Wise '98

Gegelia '98



$$V_S = C_0 + C_2 Q^2 + \ldots$$

$$V_S \frac{Q^3}{4\pi}\frac{m_N}{Q^2} V_S = V_S \left[\frac{m_N Q}{4\pi} V_S\right]$$

$$\Rightarrow = \frac{Q}{\aleph}$$

$$\Rightarrow T_{NN} \sim \frac{4\pi}{m_N M_{nuc}} \left\{ \frac{M_{nuc}}{\aleph + iQ} + \left(\frac{Q}{\aleph + iQ}\right)^2 + \ldots \right\}$$

$$\nu = -1 \qquad \nu = 0$$

p, other waves

if

$$\begin{cases} C_0 = \dfrac{4\pi}{m_N \aleph} \\ C_2 = \dfrac{4\pi}{m_N \aleph^2 M_{nuc}} \\ \vdots \end{cases}$$

but

$$\begin{cases} C_2' = \dfrac{4\pi}{m_N M_{nuc}^3} \\ \vdots \end{cases}$$

s wave

- scattering length: $a_0 \sim 1/\aleph$
- effective range: $r_0 \sim 1/M_{nuc}$
- ⇒ b.s. at $Q \sim i\,\aleph$

Alternative: auxiliary field

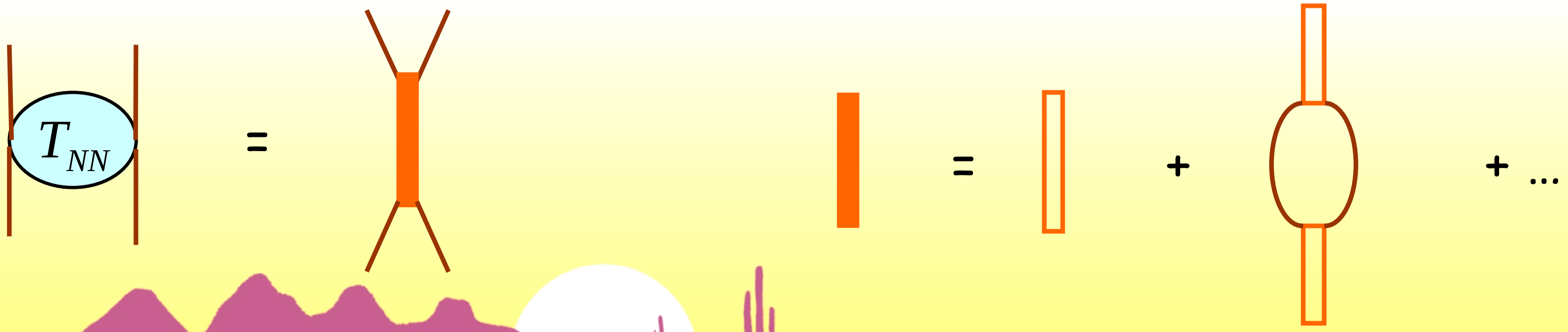
Kaplan '97
v.K. '99

$$L_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + T^{+}(-\Delta)T + \frac{g}{\sqrt{2}}\left[T^{+}NN + N^{+}N^{+}T\right]$$

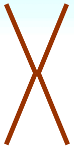
$$+ N^{+}\frac{\nabla^4}{8m_N^3}N + \underset{\text{sign}}{\sigma} T^{+}\left(i\partial_0 + \frac{\nabla^2}{4m_N}\right)T + \ldots$$

integrate out auxiliary field: same Lag as before with $C_0 = \frac{g^2}{\Delta}, \ldots$

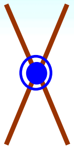
$$\Delta \sim \aleph, \quad \frac{g^2}{4\pi} \sim \frac{1}{m_N}, \quad \ldots$$



renormalization: s wave

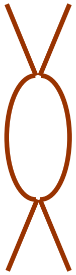


$$\sim C_0(\Lambda)$$



$$\sim C_2(\Lambda)\,k^2$$

(next order)

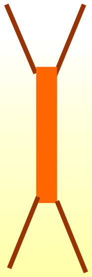


$$\sim C_0(\Lambda)^2 m_N \left[\#\Lambda + \frac{ik}{4\pi} + \#\frac{k^2}{\Lambda} + \dots \right]$$

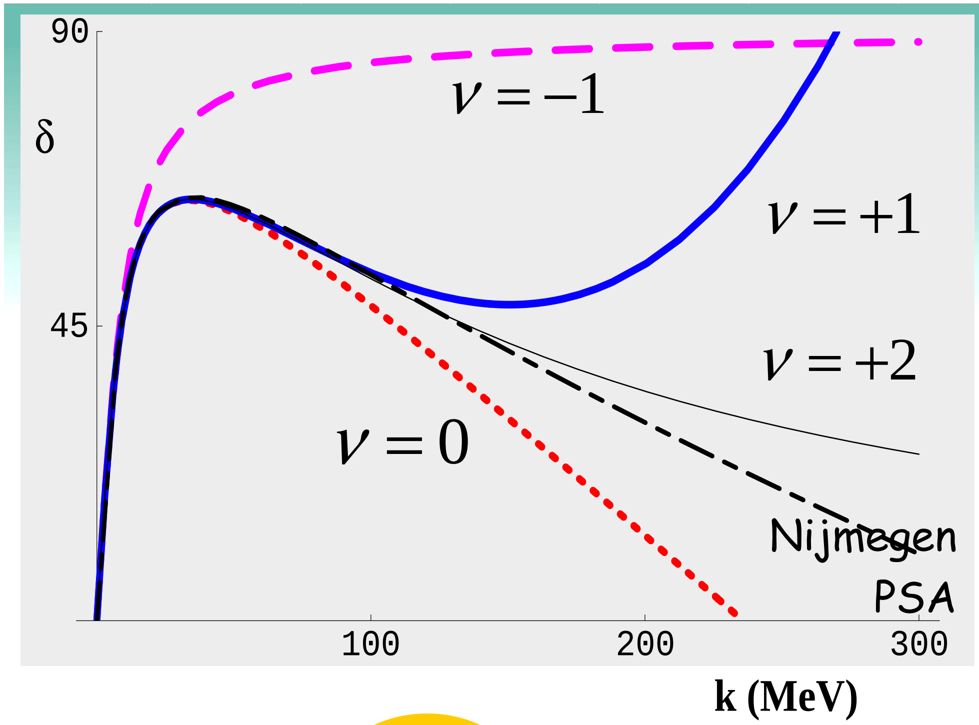
long-range physics:
non-analytic in *E*
- unitarity

short-range physics:
analytic in *E*
- high momenta in loops
- counterterms

or



$$\sim \left[\left(-\frac{\Delta}{g^2} + \#m_N\Lambda + \dots \right) + i\frac{m_N k}{4\pi} + \dots \right]^{-1}$$



Chen, Rupak + Savage '99

fitted $a_1 = 5.42\ \mathrm{fm}\ (\mathrm{exp})$

$r_1 = 1.75\ \mathrm{fm}\ (\mathrm{exp})$

predicted

$B_d = 1.91\ \mathrm{MeV}\ (\nu = 0)$

$B_d = 2.24\ \mathrm{MeV}\ (\mathrm{expt})$

S_0

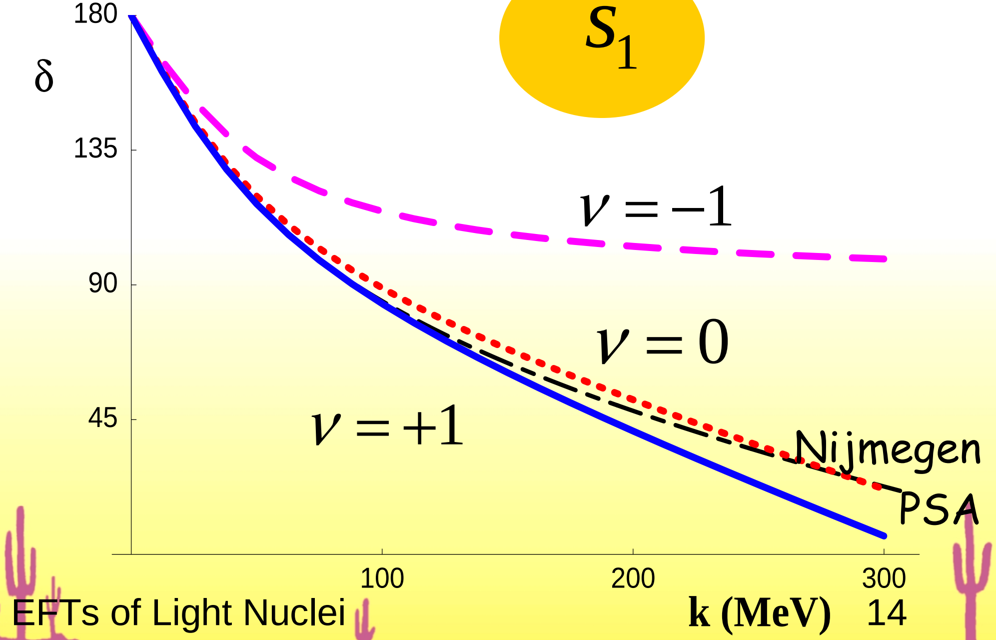
fitted $a_0 = -20.0\ \mathrm{fm}\ (\mathrm{exp})$

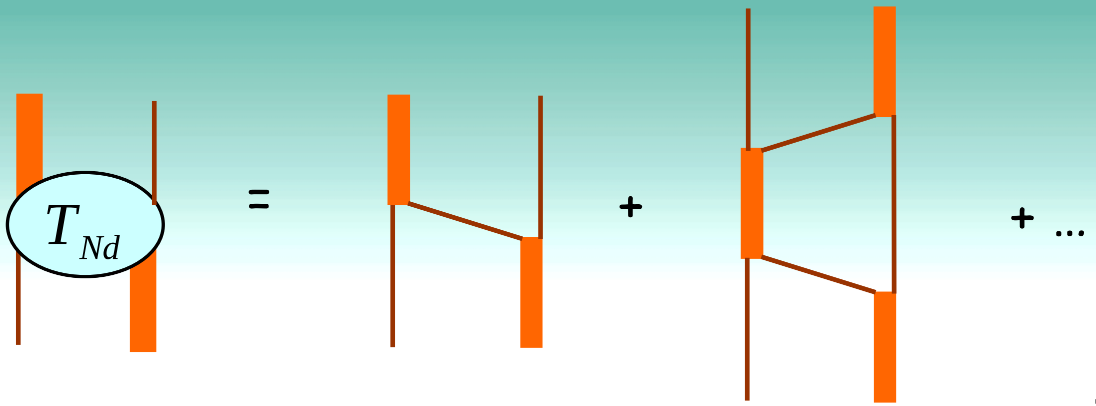
$r_0 = 2.78\ \mathrm{fm}\ (\mathrm{exp})$

predicted

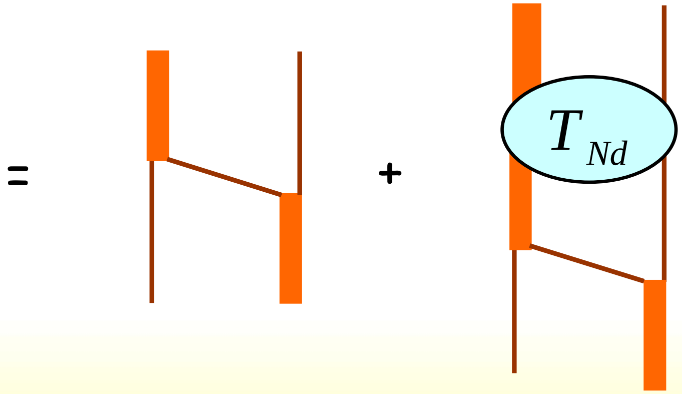
$B_{d^*} = 0.09\ \mathrm{MeV}\ (\nu = 0)$

S_1





$$\sim \frac{g^2}{Q^2/m_N} \qquad \sim \frac{Q^3}{4\pi}\left(\frac{g^2}{Q^2/m_N}\right)^2 \qquad \sim \frac{g^2}{Q^2/m_N}\,\frac{Q}{\aleph}$$



$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3}\, K_{ONE}\, T_{Nd}$$

$$L_{EFT} = \ldots + D_0\, N^+N\, N^+N\, N^+N + \ldots$$

naïve
dimensional analysis $D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{M_{nuc}^{\,3}} \quad (\nu = +1)$

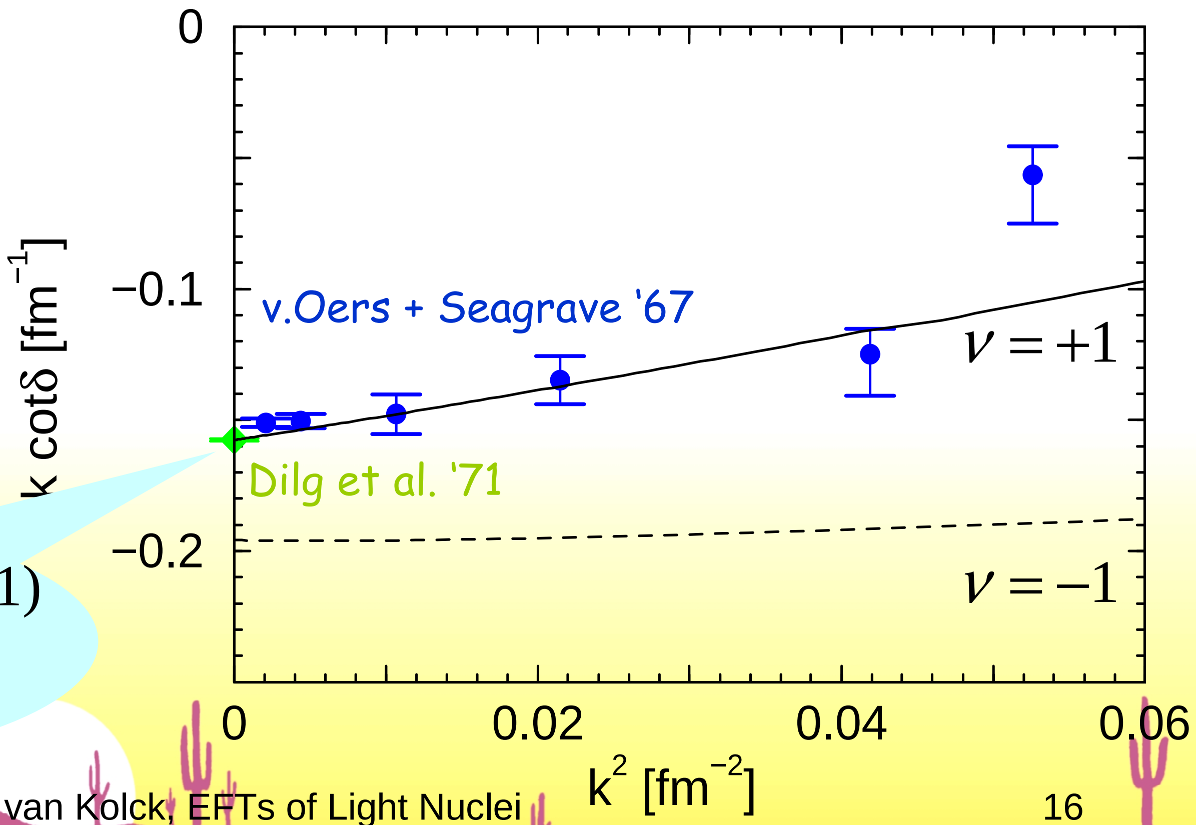
$S_{3/2}$ no three-body force up to $\nu = +3$

Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98

$$\lambda \le \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

$$T_{Nd} \xrightarrow{p >> \aleph} \frac{1}{p^2}$$

$$\Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p \sim \aleph} 0$$



predicted

$$a_{3/2} = 6.33 \pm 0.10 \text{ fm } (\nu = +1)$$

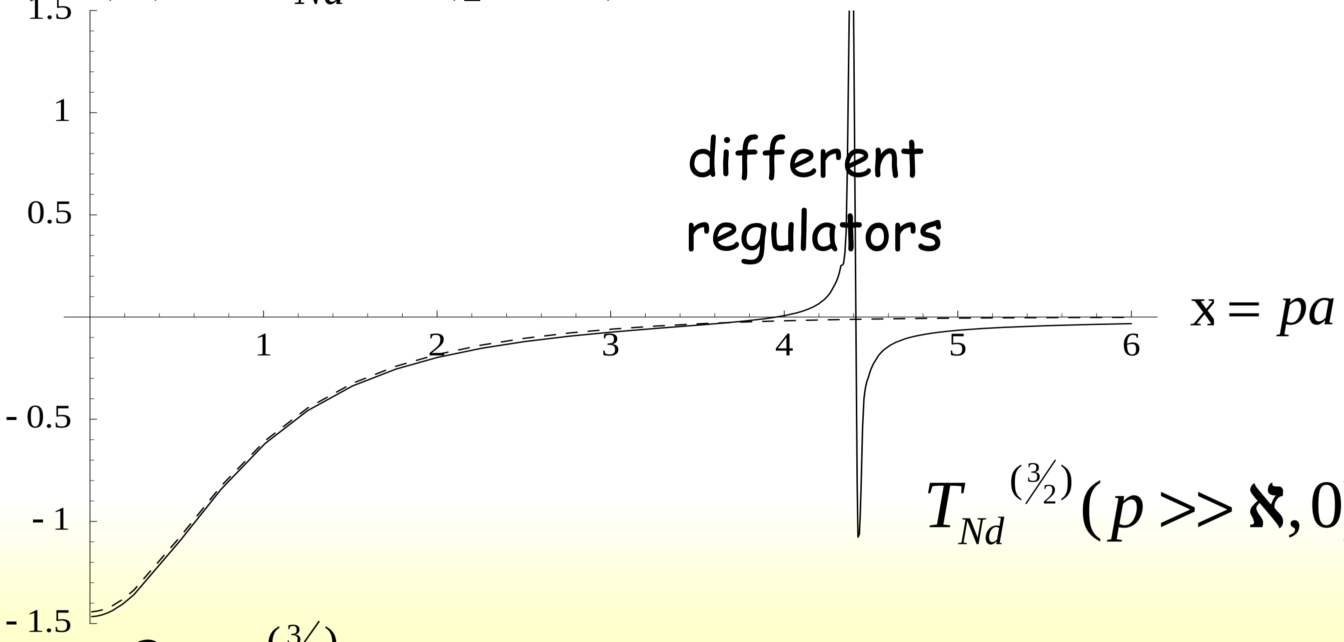
$$a_{3/2} = 6.35 \pm 0.02 \text{ fm (exp)}$$

renormalization: quartet s wave

$$\lambda \le \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

Skornyakov + Ter-Martirosian '60
Bedaque + v.K. '97
Bedaque, Hammer + v.K. '98

$$a(x) \propto T_{Nd}^{(3/2)}(pa,0)$$



$$T_{Nd}^{(3/2)}(p >> \aleph, 0) \propto \frac{1}{p^2}$$

$$\frac{\partial T_{Nd}^{(3/2)}(p \sim \aleph, 0)}{\partial \Lambda} \cong 0$$

$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03

$$T_{Nd} \xrightarrow{p>>\aleph} A\ \cos\left(s_0 \ln\frac{p}{\Lambda}+\delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial\Lambda} \xrightarrow{p\sim\aleph} \neq 0$$ **unless**

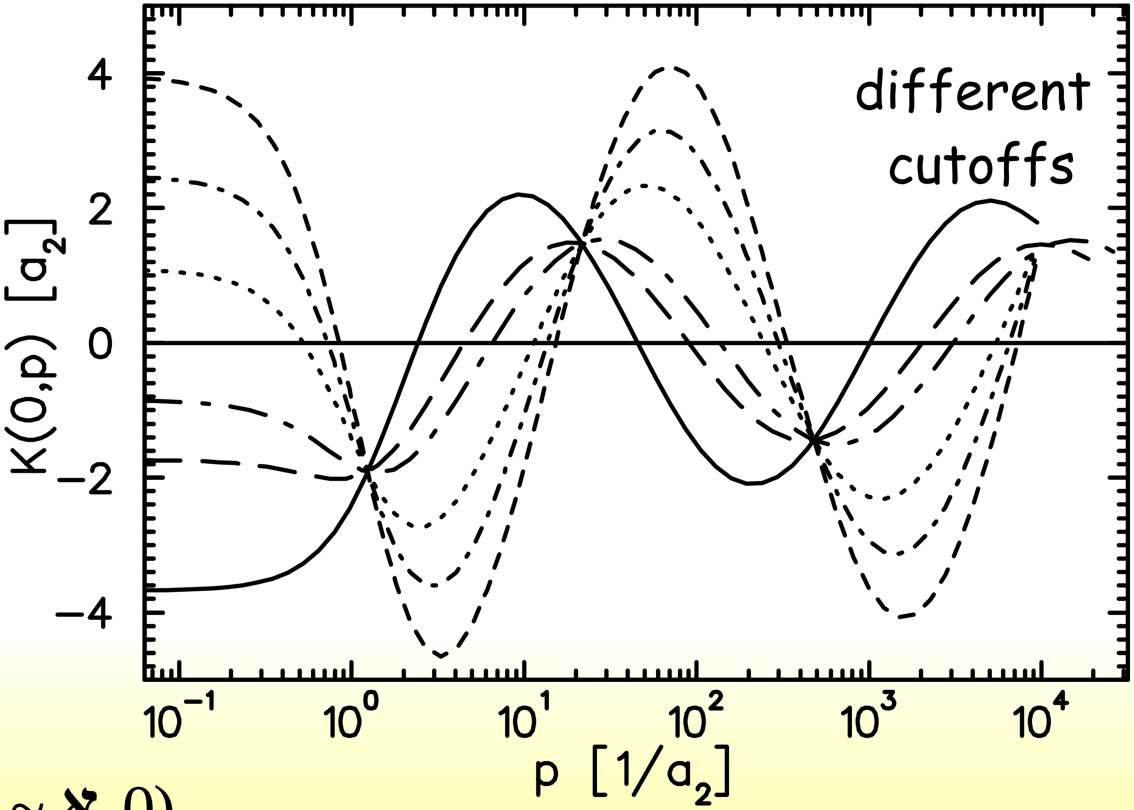
$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu = -1)$$

renormalization: doublet s wave -I

Danilov '63

Bedaque, Hammer + v.K. '99 '00

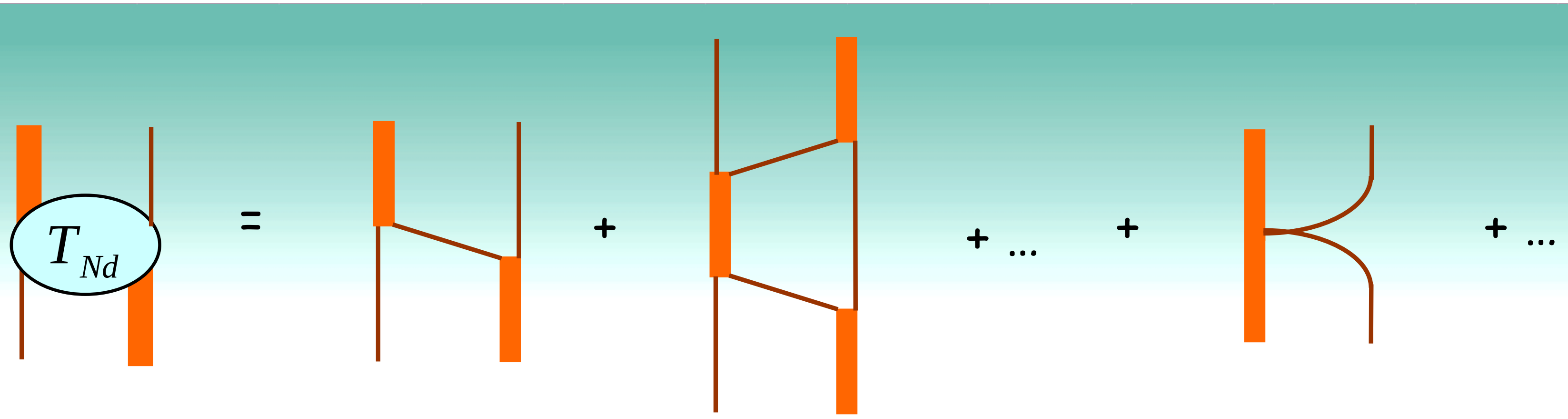
$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$



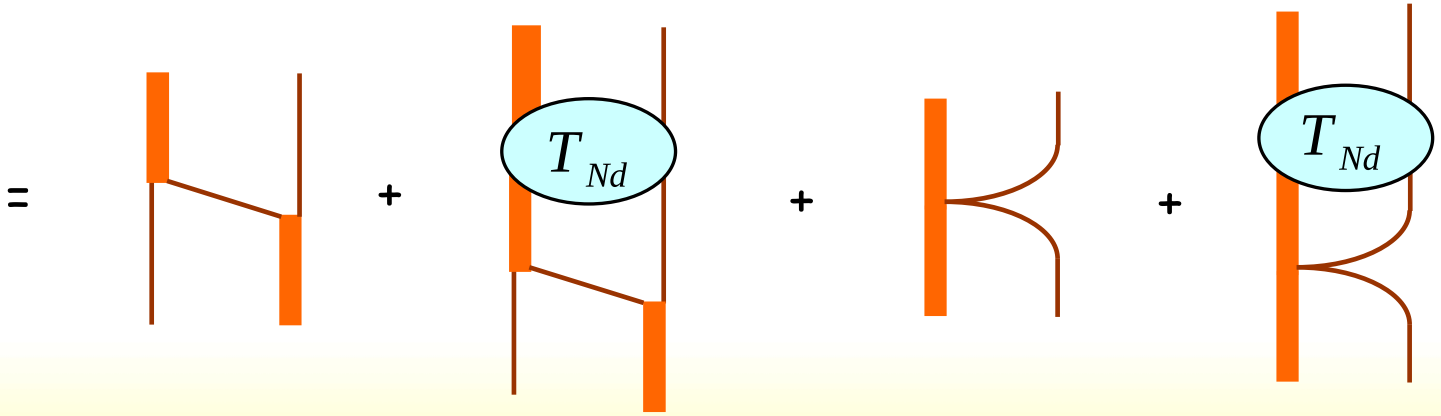
$$\frac{\partial T_{Nd}^{(\frac{1}{2})}(p \sim \aleph, 0)}{\partial \Lambda} \neq 0$$

$$T_{Nd}^{(\frac{1}{2})}(p >> \aleph, 0) \propto \cos\left(s_0 \ln\frac{p}{\Lambda} + \delta \right)$$

$$s_0 \cong 1.0024$$



$$\sim \frac{g^2}{Q^2/m_N} \qquad\qquad \sim \frac{4\pi}{\aleph^2}$$



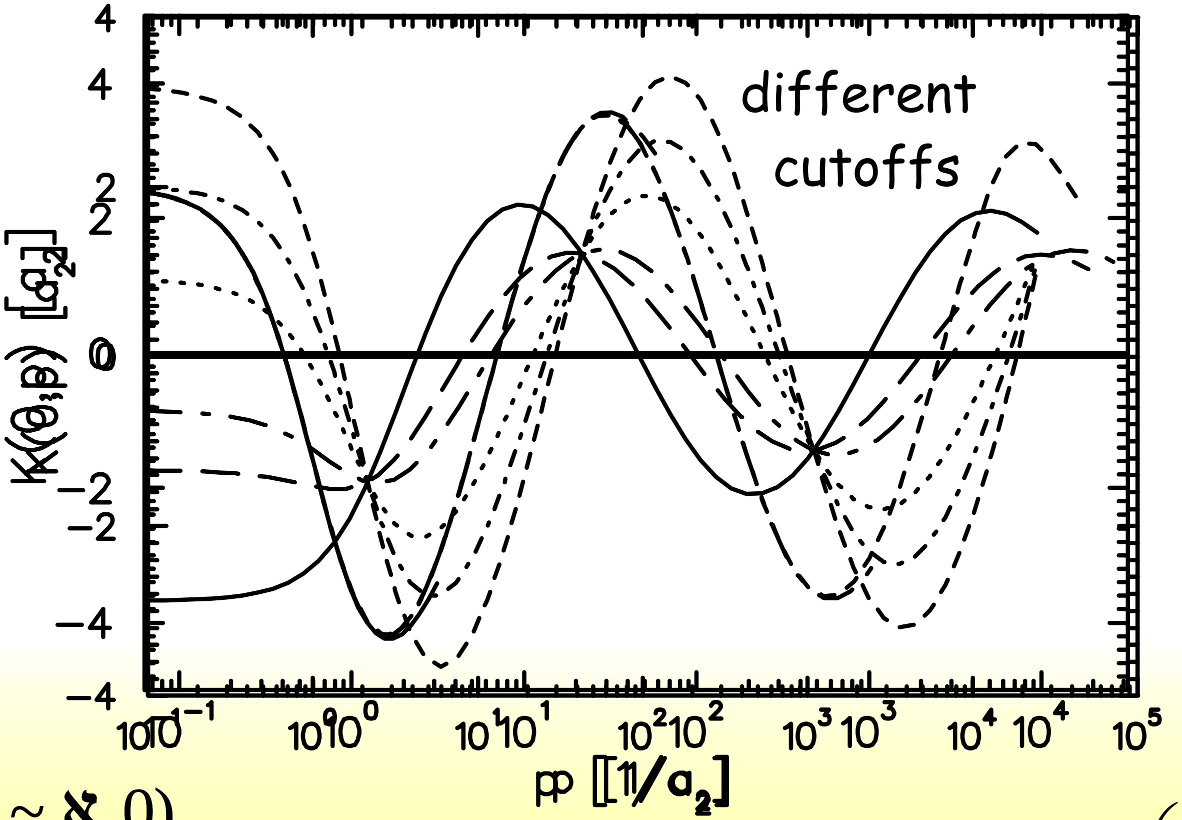
$$T_{Nd} = K_{ONE} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} K_{ONE}\, T_{Nd} + K_{TBF} + \lambda \int_0^{\Lambda} \frac{d^3 l}{(2\pi)^3} K_{TBF}\, T_{Nd}$$

renormalization: doublet s wave -I

Danilov '63

Bedaque, Hammer + v.K. '99 '00

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$



$$\frac{\partial T_{Nd}^{(\frac{1}{2})}(p \sim \aleph, 0)}{\partial \Lambda} \not\equiv 0$$

$$T_{Nd}^{(\frac{1}{2})}(p >> \aleph, 0) \propto \cos\left(s_0 \ln \frac{p}{\Lambda^*} + \delta\right)$$

$$s_0 \cong 1.0024$$

$S_{1/2}$

$$\lambda > \lambda_c \equiv \frac{3\sqrt{3}}{4\pi}$$

Bedaque, Hammer + v.K. '99 '00
Hammer + Mehen '01
Bedaque et al. '03

$$T_{Nd} \xrightarrow{p>>\aleph} A\cos\left(s_0 \ln\frac{p}{\Lambda}+\delta\right) \Rightarrow \frac{\partial T_{Nd}}{\partial \Lambda} \xrightarrow{p\sim\aleph} \neq 0$$ **unless**

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} \quad (\nu=-1)$$

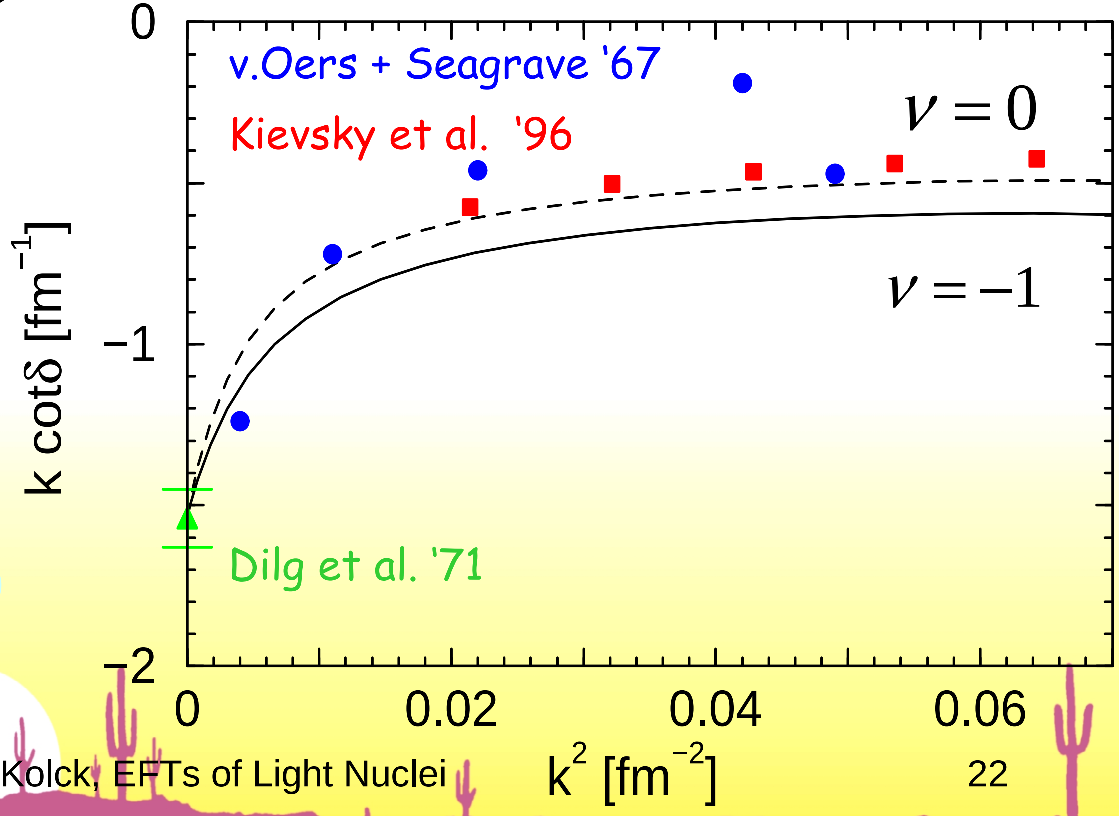
(limit cycle!)

fitted

$$a_{1/2} = 0.65\,\mathrm{fm}\,(\mathrm{exp})$$

predicted

$$B_t = 8.3\,\mathrm{MeV}\,(\nu=0)$$
$$B_t = 8.48\,\mathrm{MeV}\,(\mathrm{expt})$$

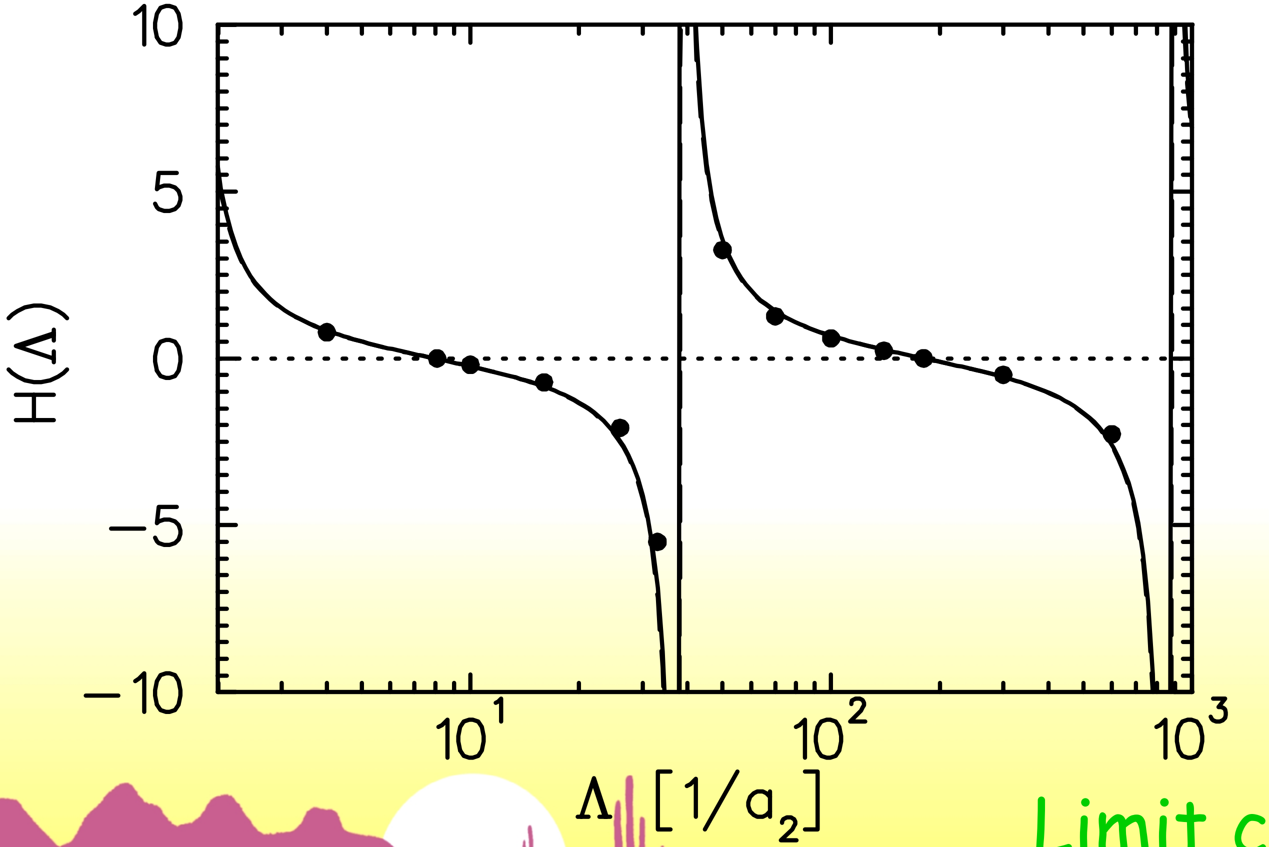


renormalization: doublet s wave -II

Bedaque, Hammer + v.K. '99 '00

$$D_0 \sim \left(\frac{4\pi}{m_N}\right)^2 \frac{1}{\aleph^2 M_{nuc}} H(\Lambda)$$

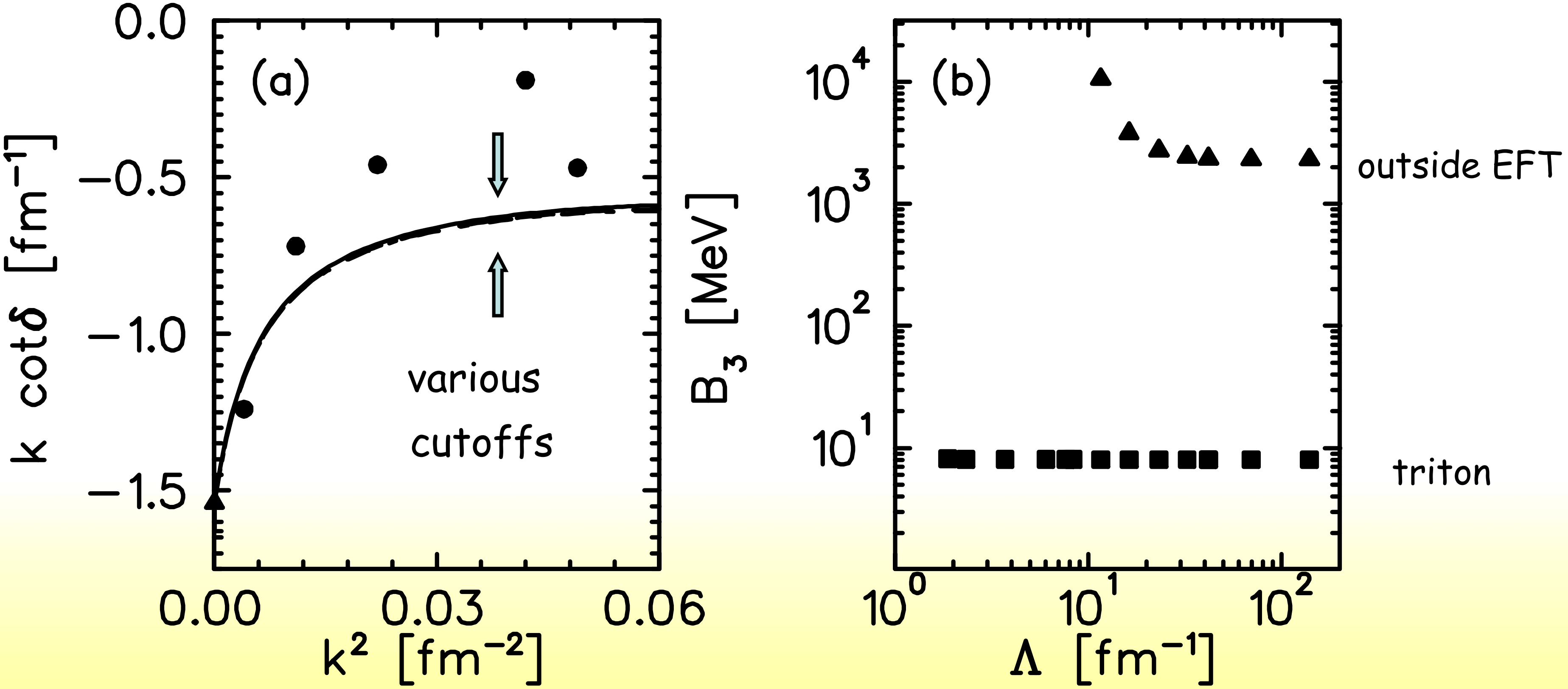
$$H(\Lambda) = \frac{\sin\left(\ln(\Lambda/\Lambda_*) + \arctan(1/s_0)\right)}{\sin\left(\ln(\Lambda/\Lambda_*) - \arctan(1/s_0)\right)}$$



Limit cycle !

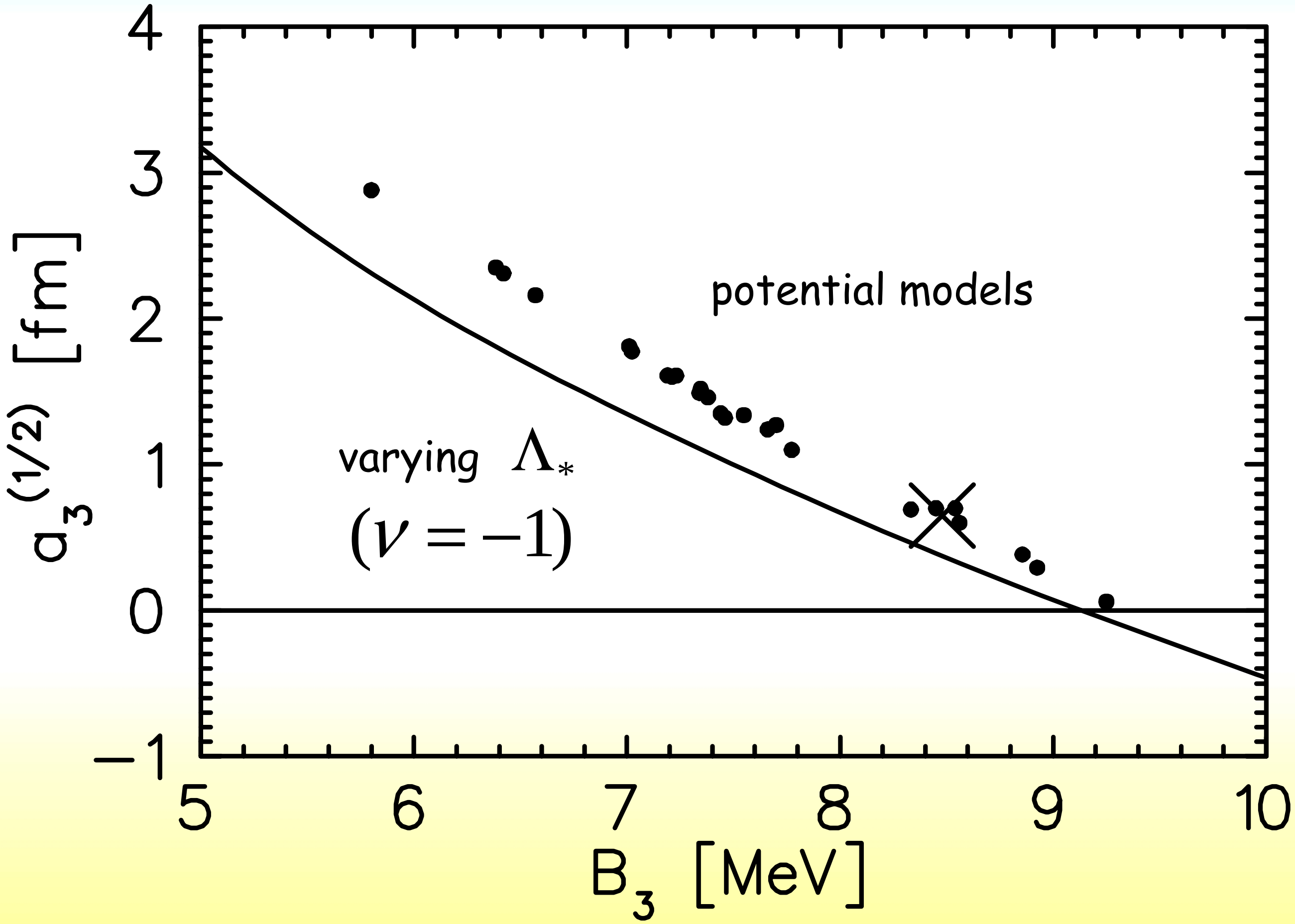
renormalization: doublet s wave -III

Bedaque, Hammer + v.K. '99 '00



Phillips line

Bedaque, Hammer + v.K. '99 '00



- many-body systems get complicated rapidly

+ (continue) focus on simpler *halo nuclei*

one or more loosely-bound nucleons (near driplines)

$$\aleph \equiv \sqrt{m_N E_N} << \sqrt{m_N E_c} \equiv M_c$$

nucleon separation energy

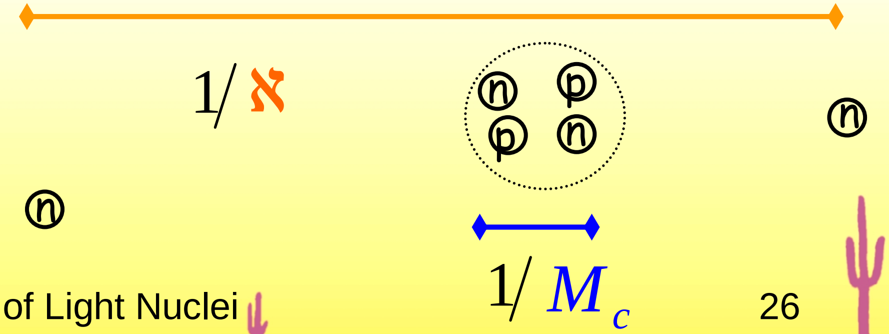
core excitation energy

e.g.

$$^4\mathrm{He} \quad \left.\begin{array}{l} B_{\alpha^*} \cong 28\,\mathrm{MeV} \\ B_{\alpha} \cong 8\,\mathrm{MeV} \end{array}\right\} E_\alpha = B_{\alpha^*} - B_\alpha \cong 20\,\mathrm{MeV}$$

$$``^5\mathrm{He}" \quad p_{3/2} \text{ resonance at } E_n \sim 1\,\mathrm{MeV}$$

$$^6\mathrm{He} \quad E_{2n} \sim 1\,\mathrm{MeV}$$



$Q \sim \aleph << M_c$

halo EFT

- degrees of freedom: nucleons, cores
- symmetries: Lorentz, $\cancel{P}$, $\cancel{T}$
- expansion in: $$\frac{Q}{M_c} = \begin{cases} Q/m_N, \quad Q/m_c \\ Q/m_\pi, \cdots \end{cases}$$

non-relativistic

multipole

simplest formulation: auxiliary fields for core + nucleon states

e.g. $${}^4\mathrm{He} \mapsto \text{scalar field } \varphi$$

$${}^4\mathrm{He} + \mathrm{N} \quad \begin{cases} s_{1/2} \equiv 0+ \mapsto \text{spin-0 field } s \\ p_{1/2} \equiv 1- \mapsto \text{spin-1/2 field } T_1 \\ p_{3/2} \equiv 1+ \mapsto \text{spin-3/2 field } T_3 \\ \vdots \end{cases}$$

Bertulani, Hammer + v.K. '02
Bedaque, Hammer + v.K. '03

$$L_{EFT} = N^{+}\left(i\partial_0 + \frac{\nabla^2}{2\,m_N}\right)N + \varphi^{+}\left(i\partial_0 + \frac{\nabla^2}{2\,m_\alpha}\right)\varphi$$

$$+ T_3^{+}\left[\sigma_3\left(i\partial_0 + \frac{\nabla^2}{2(m_\alpha + m_N)}\right) - \Delta_3\right]T_3$$

$$+ \frac{g_3}{\sqrt{2}}\left[T_3^{+}\vec{S}^{+}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

spin transition
operator

$$+ s^{+}(-\Delta_0)s + \frac{g_0}{\sqrt{2}}\left[s^{+}N\varphi + \text{H.c.}\right]$$

$$+ \dots$$

$$+ T_1^{+}(-\Delta_1)T_1 + \frac{g_1}{\sqrt{2}}\left[T_1^{+}\vec{\sigma}\cdot(N\vec{\nabla}\varphi - \varphi\vec{\nabla}N) + \text{H.c.}\right]$$

$$+ \dots$$

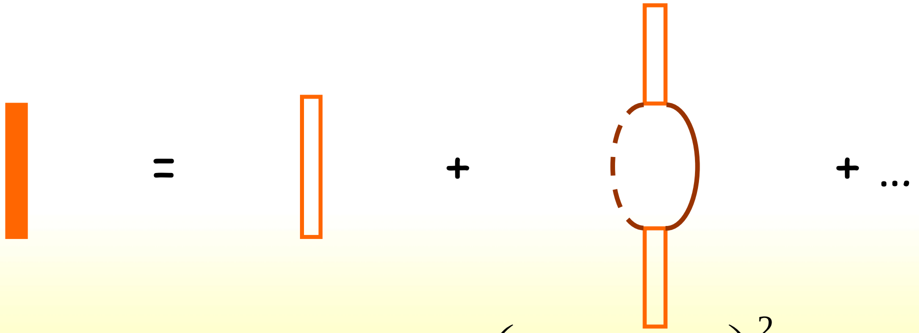
$N\alpha$ $\quad p_{3/2}$

$$= \frac{i\,\sigma_3}{E-\sigma_3\Delta_3} = \frac{i\,\sigma_3 2\mu}{k^2-\sigma_3 2\mu\Delta_3}$$

reduced mass

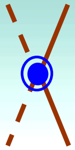
resonance at $Q \sim \pm \aleph$ if $\sigma_3\Delta_3 > 0$ and

$$\Delta_3 \sim \frac{\aleph^2}{\mu}, \quad \frac{g_3^2}{4\pi} \sim \frac{1}{\mu^2 M_c}, \ \ldots \quad \Rightarrow \quad a_{1+} \sim \frac{1}{\aleph^2 M_c}, \quad r_{1+} \sim M_c, \ \ldots$$

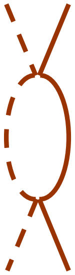


$$\sim \frac{\mu}{Q^2-\aleph^2} \quad \sim \left(\frac{\mu}{Q^2-\aleph^2}\right)^2 \frac{4\pi\,Q^2}{\mu^2 M_c}\,\frac{Q^3}{4\pi}\,\frac{\mu}{Q^2} \sim \frac{\mu}{Q^2-\aleph^2}\,\frac{Q^2}{Q^2-\aleph^2}\,\frac{Q}{M_c}$$

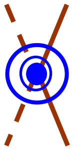
renormalization: p wave



$$\sim C_2'(\Lambda)\ \vec{p}\cdot\vec{p}\,'$$



$$\sim C_2'(\Lambda)^2\ \vec{p}\cdot\vec{p}\,'\ \mu\left[\#\,\Lambda^3+\#\,\Lambda\,k^2+\frac{i\,k^3}{12\pi}+\#\frac{k^4}{\Lambda}+\dots\right]$$



$$\sim C_4'(\Lambda)\ k^2\ \vec{p}\cdot\vec{p}\,'$$

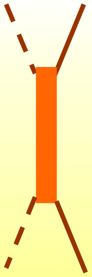
OOPS!

cannot resum C_4'

without C_6'

without … ?!

but

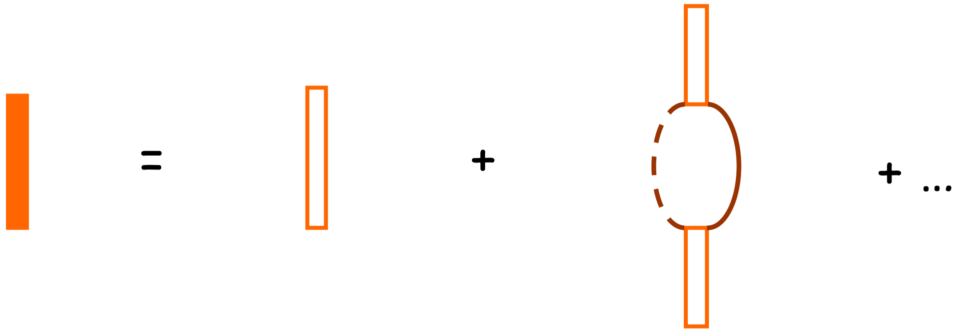


$$\sim\left[\left(-\frac{\Delta}{g^2}+\#\,\mu\Lambda^3\right)+\left(\frac{\sigma}{\mu g^2}+\#\,\mu\Lambda\right)k^2+i\frac{\mu k^3}{12\pi}+\dots\right]^{-1}\vec{p}\cdot\vec{p}\,'$$

other waves:

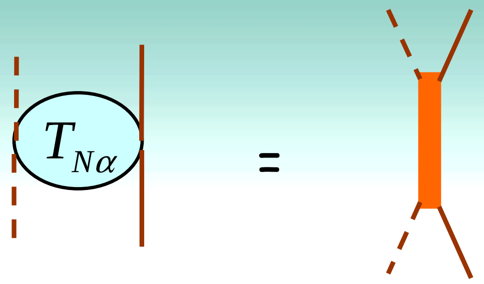
$$\Delta_0 \sim \Delta_1 \sim \ldots \sim M_c,$$

$$\frac{g_0^2}{4\pi} \sim \frac{1}{\mu}, \quad \frac{g_1^2}{4\pi} \sim \frac{1}{\mu^2 M_c}, \quad \ldots \Rightarrow \begin{cases} a_{0+} \sim \dfrac{1}{M_c}, & r_{0+} \sim \dfrac{1}{M_c}, \quad \ldots \\ a_{1-} \sim \dfrac{1}{M_c^3}, & r_{1-} \sim M_c, \quad \ldots \\ \vdots & \end{cases}$$



$$\sim \frac{1}{M_c} \qquad \sim \left(\frac{1}{M_c}\right)^2 \frac{4\pi}{\mu} \frac{Q^3}{4\pi} \frac{Q}{\mu^2} \sim \frac{1}{M_c}\frac{Q}{M_c} \qquad s_{1/2}$$

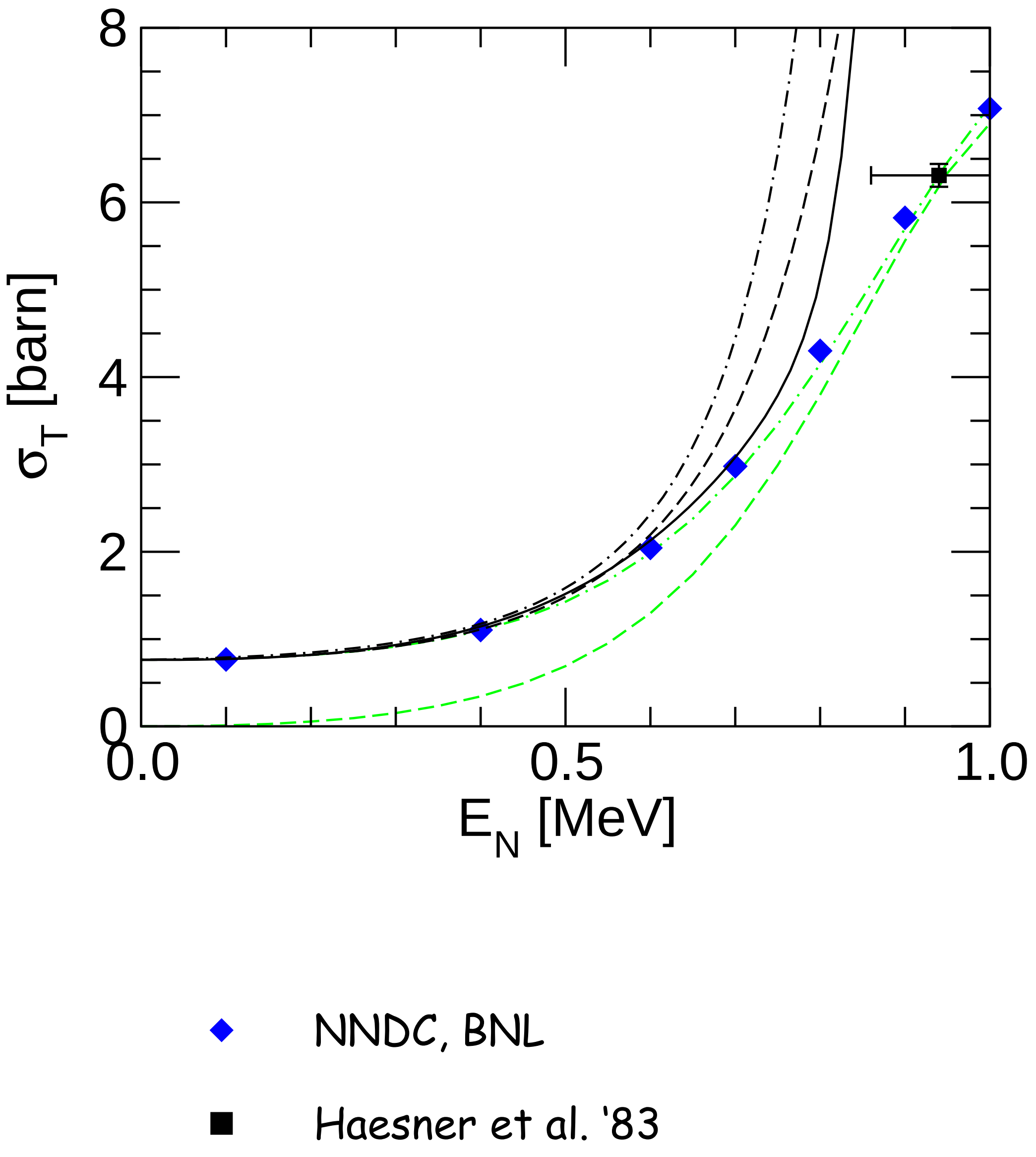
$$\sim \left(\frac{1}{M_c}\right)^2 \frac{4\pi Q^2}{\mu^2 M_c} \frac{Q^3}{4\pi} \frac{\mu}{Q^2} \sim \frac{1}{M_c}\frac{Q^3}{\mu M_c^2} \qquad p_{1/2}$$

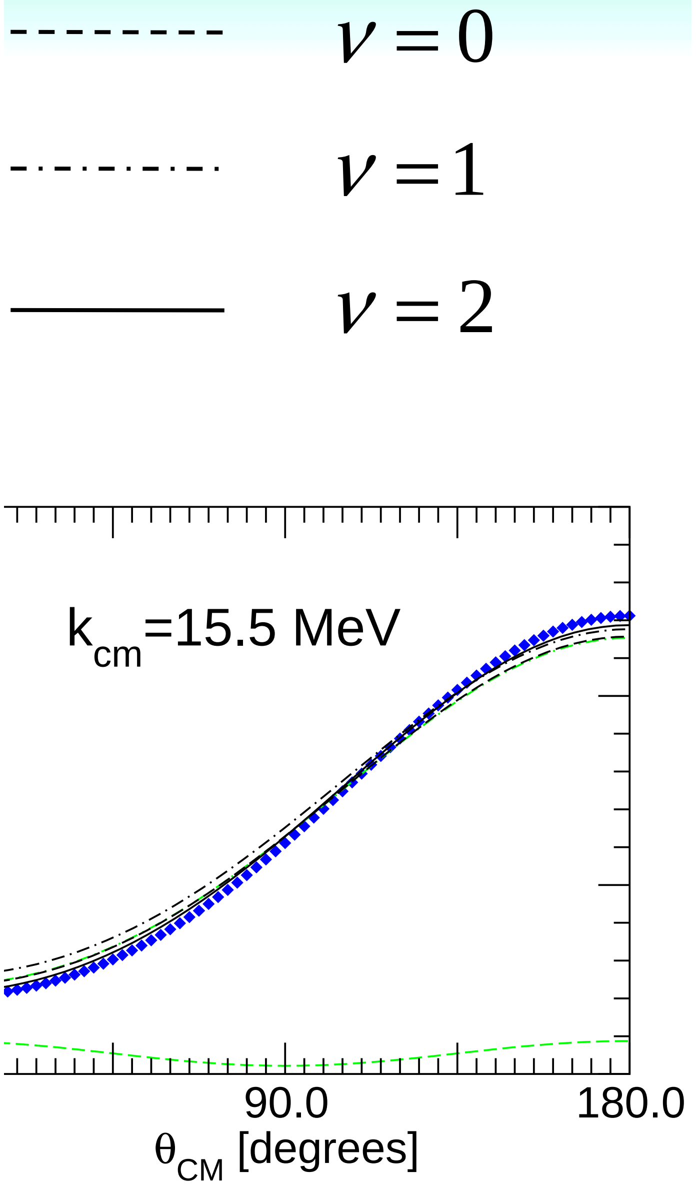


$$T_{N\alpha} \sim \frac{4\pi}{\mu M_c}\left\{\begin{array}{lllll} \frac{Q^2}{Q^2-\aleph^2} & +\frac{Q}{M_c}\left(\frac{Q^2}{Q^2-\aleph^2}\right)^2 & +\left(\frac{Q}{M_c}\right)^2\left(\frac{Q^2}{Q^2-\aleph^2}\right)^3 & +\ldots \\ 1 & + \quad 0 & + \quad \left(\frac{Q}{M_c}\right)^2 & +\ldots \\ 0 & + \quad 0 & + \quad 0 & +\ldots \\ \ldots & & & \end{array}\right\}\begin{array}{l} p_{3/2} \\ s_{1/2} \\ p_{1/2} \\ \end{array}$$

$$\nu = 0 \qquad \nu = 1 \qquad \nu = 2$$

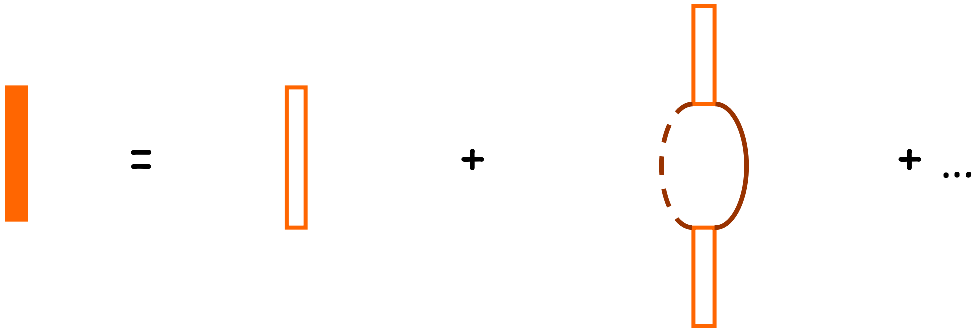
Bedaque, Hammer + v.K. '03





except at $Q = \aleph \pm O\left(\frac{\aleph^2}{M_c}\right)$ where

$p_{3/2}$



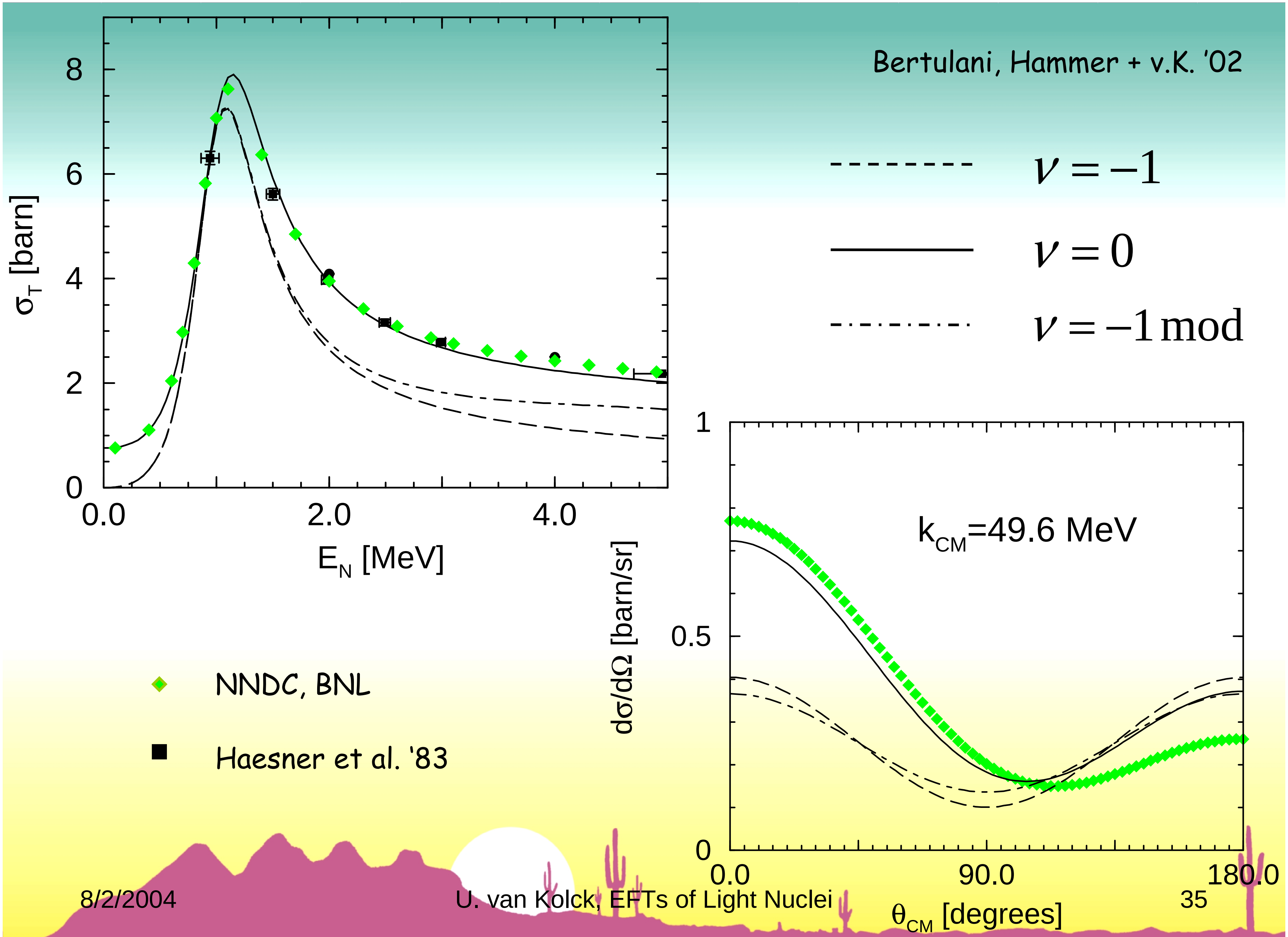
$$\sim \frac{\mu M_c}{\aleph^3} \qquad \sim \left(\frac{\mu M_c}{\aleph^3}\right)^2 \frac{4\pi\aleph^2}{\mu^2 M_c}\,\frac{\aleph^3}{4\pi}\,\frac{\mu}{\aleph^2} \sim \frac{\mu M_c}{\aleph^3}$$

⇒ enhanced by $\frac{M_c}{\aleph}$

⇒ resum self-energy

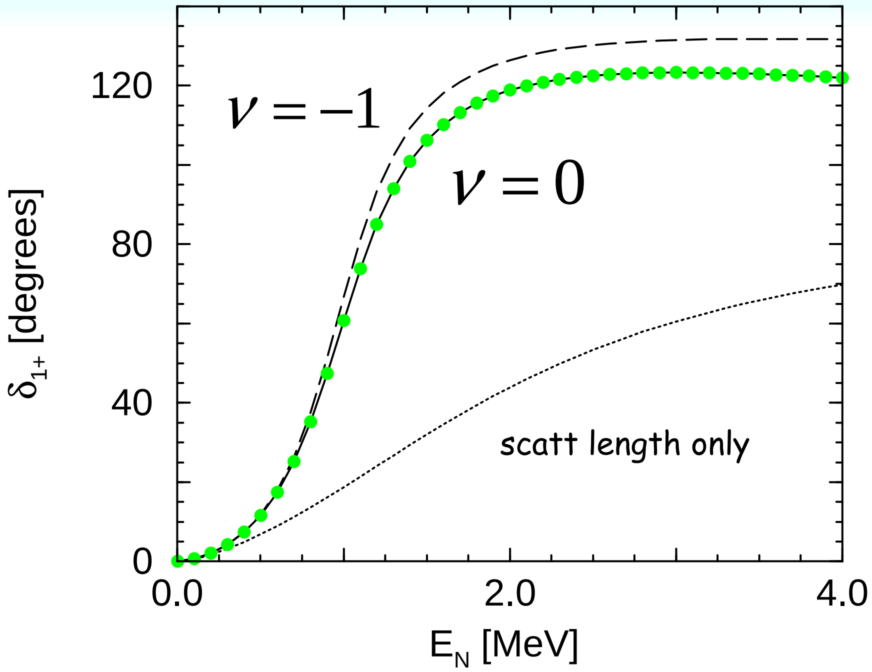
$$T \propto \frac{i\ \Gamma(E)}{E - E_0 + \frac{i}{2}\Gamma(E)}$$

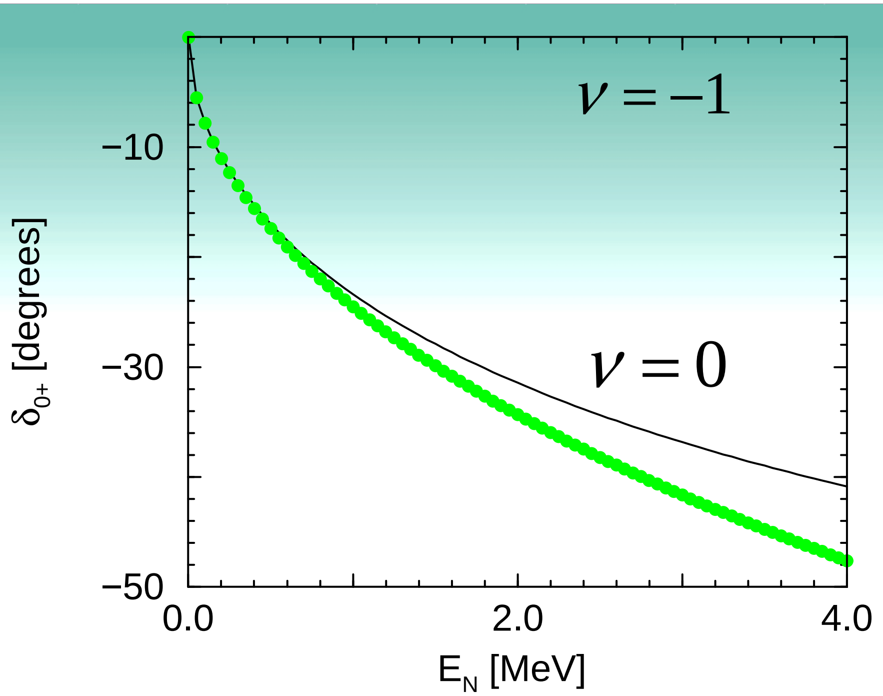
Bertulani, Hammer + v.K. '02

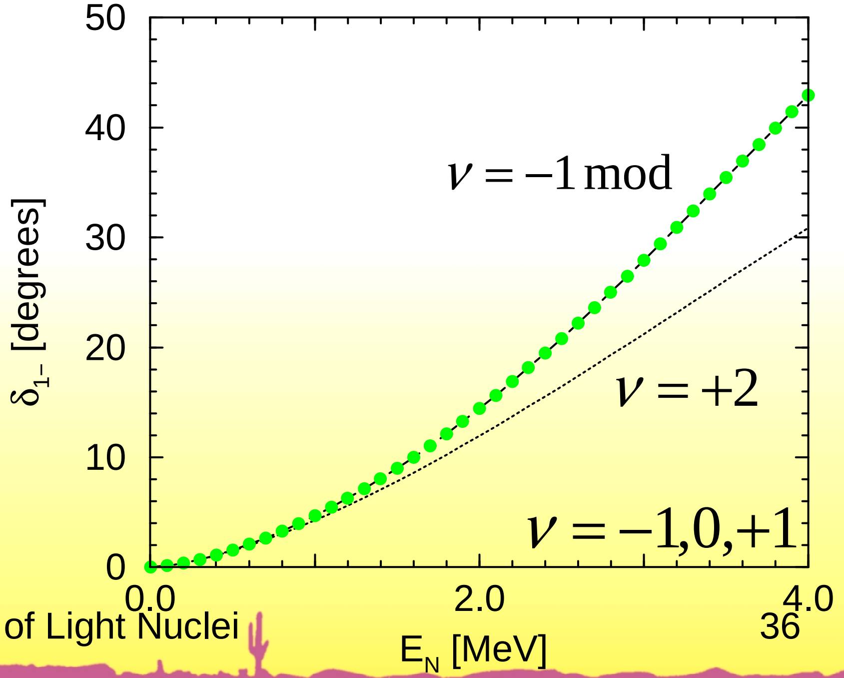


Bertulani, Hammer + v.K. '02

● PSA, Arndt et al. '73







$$E_0 \cong 0.80\ \mathrm{MeV}$$

$$\Gamma(E_0) \cong 0.55\ \mathrm{MeV}$$

Outlook

- three-body bound states:

e.g. $^{6}\mathrm{He} = \mathrm{b.s.}\left(^{4}\mathrm{He} + n + n\right)$ Hammer + v.K., in progress

[c.f. $^{3}\mathrm{H} = \mathrm{b.s.}\left(p + n + n\right)$ Bedaque, Hammer + v.K. '99]

- reactions:

e.g. $p + {}^{7}\mathrm{Be} \rightarrow {}^{8}\mathrm{B} + \gamma$

[c.f. $p + n \rightarrow d + \gamma$ Chen et al. '00]

➢ new, systematic approach to physics near drip lines