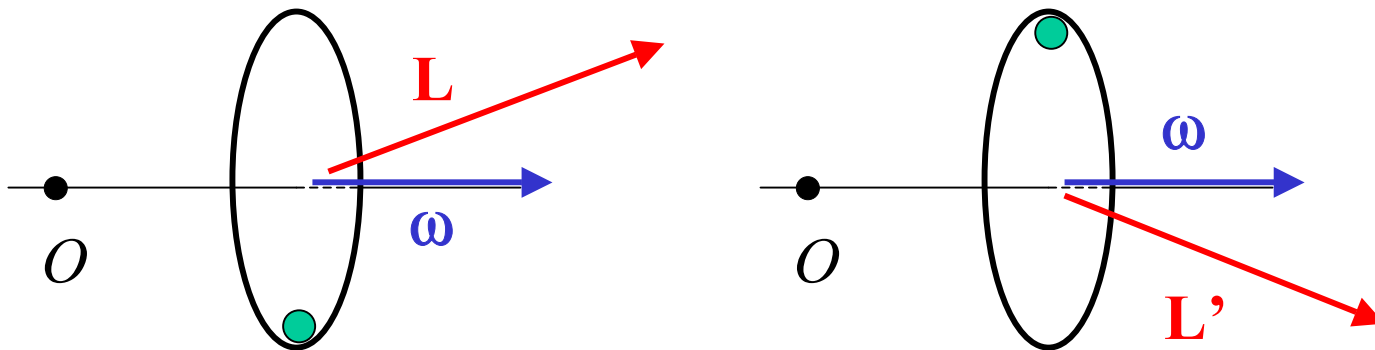


Rotation of Asymmetric Objects

- We learned last time that the relation $\mathbf{L} = I\boldsymbol{\omega}$ only holds if the rotating system is symmetric about the axis of rotation
- What happens when a system rotates about an axis that is not a symmetry axis?
- Consider a wheel with extra mass on one side, rotating about an axle at the center
- We see that \mathbf{L} changes direction as the wheel rotates:



- Since the direction of \mathbf{L} is different, a torque must have been applied to the wheel
 - In other words, the axle exerts a force at the point of attachment to the wheel
- In practical terms, this is bad news, as this force will increase the wear on the axle/wheel joint
 - This is why automobile wheels are carefully balanced using small weights before installation
- Note that the unbalanced wheel cannot rotate freely about its center – if the axle is not present, it would rotate about a point closer to the extra mass

Conservation of Angular Momentum

- We saw last time that the change in angular momentum was given by:

$$\frac{d\mathbf{L}}{dt} = \boldsymbol{\tau}_{\text{ext}}$$

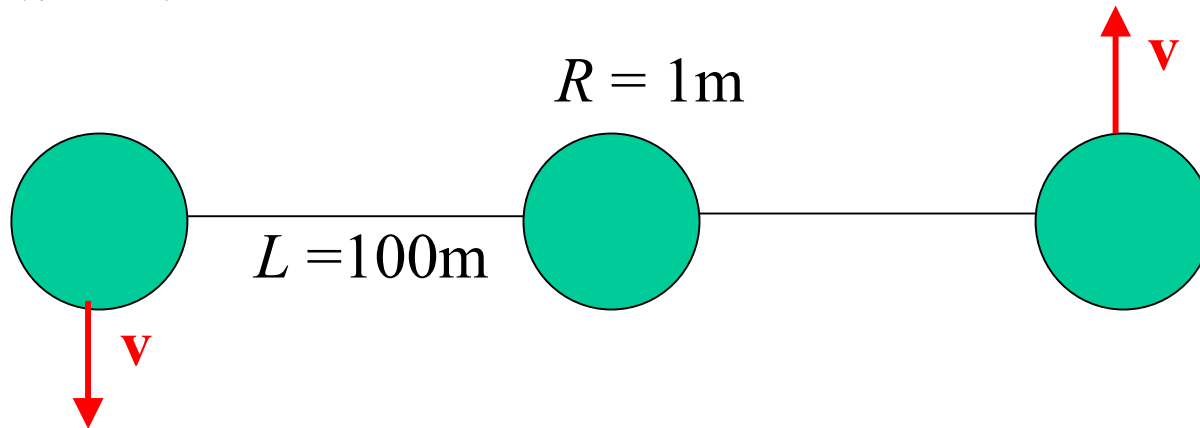
- One consequence of this is that if there is no external torque, the angular momentum is constant
 - In other words, angular momentum is conserved
- We know that every symmetry of the laws of physics corresponds to a conserved quantity
 - e.g., momentum conservation is a consequence of the fact that the laws of physics don't depend on position, and energy conservation is a consequence of the fact that they don't depend on time
- Angular momentum is conserved due to the rotational invariance of the laws of physics

Examples of Angular Momentum Conservation

- Athletes in several sports take advantage of conservation of angular momentum
- A figure skater begins a spin with arms extended
 - The torque applied by the ice on one of his skates provides angular momentum
- Since the skater is (nearly) symmetric about his axis of rotation, $\mathbf{L} = I\boldsymbol{\omega}$
- Next he brings his arms in toward his chest
 - i.e., the mass of his arms moves much closer to his axis of rotation
 - This significantly reduces I
- Since I decreases, and \mathbf{L} is conserved, $\boldsymbol{\omega}$ must increase
 - The skater spins faster

Example 2: Retrieving a Satellite

- An important satellite is malfunctioning. NASA decides to send up the space shuttle to collect the satellite and bring it back to Earth for repairs. The satellite consists of three identical cylindrical modules, and is rotating at a leisurely 1 rev/min:



- In order to fit in the cargo bay, the two outlying modules have to be “reeled in”
- Given that the central module has motors that can do this, is there a problem?

- In the initial configuration, the moment of inertia about the axis of rotation was:

$$\begin{aligned} I &= \frac{1}{2}mR^2 + 2\left(\frac{1}{2}mR^2 + m(L + 2R)^2\right) \\ &= m\left(\frac{3}{2} \cdot (1\text{m})^2 + 2 \cdot (102\text{m})^2\right) = m \cdot 20900\text{m}^2 \end{aligned}$$

- After the outer modules are pulled in, the moment of inertia becomes:

$$\begin{aligned} I' &= \frac{1}{2}mR^2 + 2\left(\frac{1}{2}mR^2 + m(2R)^2\right) \\ &= m\left(\frac{3}{2} \cdot (1\text{m})^2 + 2 \cdot (2\text{m})^2\right) = m \cdot 9.5\text{m}^2 \end{aligned}$$

- Since there was no external torque applied during the process, L remains constant throughout
 - The satellite is symmetric about the axis of rotation, so $L = I\omega$
- Therefore, we have:

$$I\omega = I'\omega'$$

$$\omega' = \omega \frac{I}{I'} = \frac{2\pi \text{rad}}{60\text{s}} \cdot \frac{m \cdot 20900\text{m}^2}{m \cdot 9.5\text{m}^2} = 230\text{rad/s}$$
$$= 37\text{rev/s}$$

- It's going to be very difficult to place such an object in the shuttle!
 - The solution is to equip the outer modules with small thrusters that can stop their rotation before they're pulled in

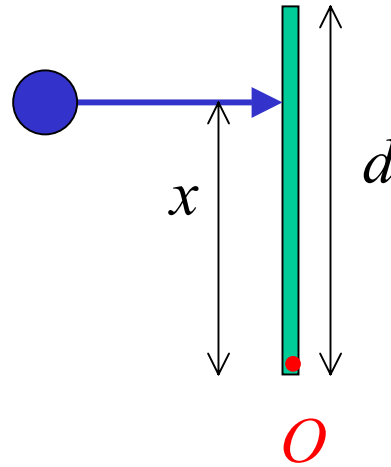
Angular Impulse

- In one of your homework problems, you will show that there is an angular analogue to impulse:

$$\tau\Delta t = \Delta L$$

- Using this concept we can solve the following problem:
- A softball player wants to hit a ball without feeling a sting in her hands
- Assuming the bat is a uniform rod, and is held at one end, how far down the bat should the ball strike?

- The situation looks like this:



- We want the point at O to remain stationary, even if there is no external force from the batter's hands
- In other words, we want:

$$\omega = \frac{v_{\text{cm}}}{x_{\text{cm}}} = \frac{2v_{\text{cm}}}{d}$$

- From the linear impulse, we know that:

$$F \Delta t = Mv_{\text{cm}}$$

- Similarly the angular impulse tells us that:

$$\tau \Delta t = xF \Delta t = I \omega$$

$$xMv_{\text{cm}} = \frac{1}{3}Md^2\omega$$

- Plugging in our desired value for ω we find that:

$$xMv_{\text{cm}} = \frac{1}{3}Md^2 \left(\frac{2v_{\text{cm}}}{d} \right) = \frac{2}{3}Mdv_{\text{cm}}$$

$$x = \frac{2}{3}d$$