GLOBAL TRANSMISSIVITY AND DIFFUSE FRACTION OF SOLAR RADIATION FOR CLEAR AND CLOUDY SKIES AS MEASURED AND AS PREDICTED BY BULK TRANSMISSIVITY MODELS

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Abstract—Measurements of cloudiness and of global, direct, and diffuse radiation taken over a 13 mo period at Davis, CA, are analyzed in terms of global transmission ($K_0$) and diffuse fraction ($K_d$) for clear sky conditions and for various cloudiness conditions. A number of global transmission clear sky models are compared with observations for ranges of total water column and turbidity and some are found to give representative values for the global radiation at the ground.

The dependence of the diffuse fraction on global transmission is found to be best represented by linear formulae—with different dependencies found for clear and cloudy conditions.

Global transmission models are also compared with observations for cloudy conditions and found to give representative values of cloud transmissivities if climatological differences in the cloudiness at the measurement site and those sites used to calibrate the cloud models are considered.

These results support the use of routine instantaneous surface meteorological data to calculate the most likely instantaneous global and diffuse radiation on a horizontal surface in the absence of any radiation measurements. These calculated irradiances are best used for solar energy system dynamic modeling in which system responses to typical sequences in meteorological conditions are being examined.

1. INTRODUCTION

With their 1960 paper, Liu and Jordan[1] introduced the notion that the ratio of diffuse to global radiation ($K_d$) incident on a horizontal surface should be a well-behaved function of the global transmissivity ($K_t$) of the atmosphere. This latter parameter, often referred to as either the clearness index or the cloudiness index, is the ratio of the actual global radiation to the extraterrestrial radiation incident on a horizontal plane. This parameterization is very attractive for estimating or modeling the response of solar energy utilization systems to varying atmospheric conditions. The major problems with the Liu and Jordan formations are: that $K_t$ must be determined in some manner; that $K_t$ is assumed to vary primarily due to cloudiness; and that the relation between $K_t$ and $K_d$ is independent of cloud type.

Dynamic model simulations of solar energy systems or structures often require instantaneous values of the radiation to drive them and to evaluate system or structure response to complex demand cycles or varying meteorological conditions. Representative models for calculating $K_t$ and $K_d$ are needed to perform these simulations. This paper presents a brief overview of ways to obtain realistic instantaneous values of $K_t$ in the absence of measured global radiation and presents a set of linear relationships between $K_t$ and $K_d$ valid for clear skies and three ranges of cloudiness, at least in Davis, CA.

2. CLEAR SKIES

2.1 General, clear sky transmittance models

The general form of the parameterized clear sky direct radiation incident on a horizontal surface at the ground is

$$I_r = S' \cos Z T_a T_w T_R T_R T_p,$$

(1)

where $S'$ is the extraterrestrial solar irradiance corrected for earth sun distance, $Z$ is the local solar zenith angle, and the $T$s are transmissivities in the atmosphere after the extinction due to processes identified by subscript. The subscripts are, a for absorption by the fixed gases, w for absorption by water vapor, R for Rayleigh scattering by fixed gases, $R_2$ for Rayleigh scattering by variable gases and p for Mie scattering by particles. The diffuse radiation is then approximated by

$$I_d = f_1(I_{w1} - I_r) + f_2(I_{w2} - I_r) + f_3.$$

(2)

In this expression $f_1$ is the forward scatter fraction from Rayleigh scattering, $f_2$ is the forward scatter fraction for Mie scattering, and $f_3$ is a correction to account for surface albedoes other than 0.25. $I_{w1}$ and $I_{w2}$ are the direct radiative components in the absence of Rayleigh scattering and Mie scattering, respectively.

The difference among models lies in how these
transmittances and forward scatter ratios and corrections are defined.

From eqns (1) and (2) the global radiation can be calculated, i.e.

$$G = I_r + I_t,$$  \hspace{1cm} (3)

as well as the global transmissivity

$$K_t = \frac{G}{S^* \cos Z},$$  \hspace{1cm} (4)

and the diffuse fraction

$$K_d = \frac{I_t}{G}. \hspace{1cm} (5)$$

The precipitable water \(y\) (cm) can be measured or approximated by

$$\gamma = 0.054 - 0.0088m + 1.08 \times 10^{-3}m^2 - 5.1 \times 10^{-5}m^3. \hspace{1cm} (11)$$

\(\beta\) is Angstrom's turbidity coefficient which falls in the range 0.01–0.3. The diffuse parameterization combines \(f_1, f_2, I_w1\) and \(I_w2\) giving

$$I_w = (0.5 + 0.3\beta) \cos^{1/3} Z(I_w - I_r)$$  \hspace{1cm} (12)

with

$$f_3 = (0.98 + 0.1\alpha_s + 0.36\beta(\alpha_s - 0.25))(I_r + I_t) - (I_r + I_t), \hspace{1cm} (13)$$

then

$$I_r = I_t + f_3$$  \hspace{1cm} (14)

and

$$G = I_r + I_t. \hspace{1cm} (15)$$

Note that in this model \(I_w\) is the direct component if no scattering had occurred and the Rayleigh forward scattering fraction (0.5) and the Mie forward scattering fraction (0.3\(\beta\)) are combined as one factor.

2.2 Clear sky RSC model

This model is based on the summaries contained in the texts by Robinson[2] and Sellers[3]. It attempts to be universal in that the transmissivities are defined in terms of basic properties such as aerosol mass and water vapor path in a Beer's law formulation, and stems from the early analyses by Angstrom.

The transmissivities in the RSC model are defined as

$$T_u = 10^{-0.022m}, \hspace{1cm} (6)$$

$$T_w = 10^{-0.066(1 - 0.01y)m}, \hspace{1cm} (7)$$

$$T_{R1}, T_{R2} = 10^{-ym}, \hspace{1cm} (8)$$

$$T_p = 10^{-0.666y(m^2)}. \hspace{1cm} (9)$$

Here \(m\) is the air mass \(= (p/p_o)m_r\), with the relative air mass computed from either the formula suggested by Kasten[9] or by Rodgers[10]

$$m_r = \frac{1}{(\cos Z + 0.15(93.885 - Z))^{-1.255}} \hspace{1cm} (\text{Ref. [9]})$$

or

$$m_r = \frac{35}{(1224\cos^2 Z + 1)^{1/2}} \hspace{1cm} (\text{Ref. [10]}).$$

The recent literature contains four such models which contain elements from numerous precursor studies. The models examined are ones developed by this author based on the summaries in Robinson[2] and Sellers[3], to be referred to hereafter as the RSC model; one by Davies, Schertzer, and Nunez[4], hereafter referred to as the DSN Model; one described by Davies and McKay[5], the MAC model; and the global model of Atwater and Ball[6], identified as the ABG model. The model adapted by Ideriah[7] from the work of King and Buckius[8] is not included as it contains a number of errors in formulation as published and does not give reasonable results. The intent here is not to try to present a comprehensive review of such models but to compare examples of models with each other and each with observations. All of these models are empirical in character although some attempt a more fundamental formulation so as to be less site specific.

2.3 The DSN model

The major difference between this and the RSC model is that the transmissivities are defined by algebraic or polynomial expressions rather than the Beer's law form of exponentials. The transmissivities are

$$T_{R1} = 1 - 0.225ym, \hspace{1cm} (16)$$

$$T_{R2} = 0.972 - 0.08262m + 0.00933m^2$$

$$- 0.0095m^3 + 0.000437m^4, \hspace{1cm} (17)$$

$$T_w = 1 - 0.077(y(m^2))^{0.3}, \hspace{1cm} (18)$$

$$T_p = k^m. \hspace{1cm} (19)$$
In this model absorption by the fixed gases (O₂, O₃ etc.) is not explicitly included—unless Tₐ is implicitly contained in the definition of Tₑ. The k in the particle transmission relation is reported to range from 0.88 to 0.98. If k = 10⁻⁶₆₆ as in the RSC model, this would represent a range of β from 0.1 to 0.01.

The diffuse component includes only the Rayleigh forward scatter fraction (0.5) with no air mass correction, i.e.

\[ f₁ = 0.5S' \cos Z (I_w - I_t) \]  

and \( f₂ = 0 \).

### 2.4 The MAC model

In this model, as in all of the others, particle absorption is not explicitly calculated but is implicit in the empirically determined particle transmittances. Like the DSN model, the transmittances are polynomial rather than exponential expressions involving the pathlengths and air mass parameters. These formulas are

\[ T_a = \frac{0.1082 \chi}{(1 + 13.86 \chi)^{0.865}} + \frac{0.00685 \chi}{1 + (10.36 \chi)^3} + \frac{0.002118 \chi}{1 + 0.0042 \chi + 3.23 \times 10^{-6} \chi^2} \]  

\[ \chi = m_U \omega, \]  

\[ U_0 = 3.5m \text{ (ozone)}, \]  

\[ T_w = 1 - \frac{0.29ym}{(1 + 141.5ym)^{0.635} + 5.925ym}, \]  

If y is not measured

\[ y = \exp[2.2572 + 0.05454\tau_d] \left( \frac{P}{p_0} \right)^{0.34} \left( \frac{\tau_0}{\tau} \right)^{1/2} \]  

where \( \tau_d \) is the surface dew point temperature and \( \tau \) the air temperature. The Rayleigh scattering transmission is tabulated as a function of m and can be calculated using

\[ T_{R₁} \cdot T_{R₂} = 1 - 0.0799m + 0.061m^2 - 0.00023m^2 + 3.33 \times 10^{-6}m^4. \]  

For the diffuse calculation \( f₁ \) is 0.5, \( f₂ \) is tabulated vs zenith angle (in degrees) and \( f₃ \) is zero. A polynomial fit to the tabulated values of \( f₂ \) gives

\[ f₂ = \max \left\{ \frac{0.922 - 1.29 \times 10^{-4}Z + 5.38}{\text{or}} \right\} \times 10^{-6}Z^2 - 7.54 \times 10^{-7}Z^3 + 9.6 \times 10^{-10}Z^4 \]  

or

\[ f₂ = 0.6 \]  

### 2.5 The ABG model

Since this model is derived to predict only global radiation its form is slightly different from the others and the transmission functions include the diffuse component. If we define the product of all the effective transmittances as \( T_e \) then the ABG model for the global radiation (irradiance) is

\[ G = S' \cos Z T_e \left( \frac{1}{1 - r_s r_a} \right). \]  

Here \( r_s \) is the surface albedo and \( r_a \) is the atmospheric reflectance, which for clear skies is 0.0685. The formulation of this model draws primarily on the summary in Kontratyev’s text[11]. The transmittances are

\[ T_e T_{R₁} T_{R₂} = 1.021 - 0.084[(m(949p \times 10^{-7} + 0.051))^{1/2}, \]  

where \( p \) is the station pressure in kPA.

\[ T_w = 1 - \frac{2.9ym}{(1 + 141.5ym)^{0.635} + 5.925ym}, \]  

\[ T_p = e^{-dU_p}. \]  

Here \( d \) is the particle extinction coefficient and \( U_p \) is the particle pathlength, two parameters not normally known in a given situation.

### 2.6 Model comparisons

It is quite obvious that all of these models differ somewhat in basic formulation and in the empirically determined transmissivity. Note that the transmission after water vapor absorption in the MAC and ABG models is identical and that \( T_p \) in the RSC, DSN, and MAC models is equivalent. Given the similarity with \( T_e \) defined in ABG, one can reasonably replace \( e^{-bU_p} \) with the k as defined in the other models. The danger in doing so in this model is that \( T_p \) should be increased by the diffuse fraction that particles scatter downward. However, this would appear to be less of an error than the assumption[6] that \( T_p = 1 \).

With the adoption of a common formulation for \( T_p \), i.e. that in the RSC model, each of these models could be used to calculate global radiation as a function of zenith angle for various values of \( \beta \) and \( y \) and therefore, predict values of \( K_t \). The diffuse frac-

<table>
<thead>
<tr>
<th>Precipitable water (y)</th>
<th>Turbidity coefficient (β)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>0.5</td>
<td>0.05</td>
</tr>
<tr>
<td>1.0</td>
<td>0.01</td>
</tr>
</tbody>
</table>
tion from the first three models can also be computed. These calculations were made for the ranges of $\beta$ and $\gamma$ shown in Table 1. These values were chosen as representative of the turbidity and precipitable water expected at Davis, CA (38.5°N; 121.1°W; 18 m MSL) for which data sets containing clear sky global direct and diffuse radiation are available.

2.7 Comparison data

The data set from Davis comprises 13 mos (1 February 1981 through 21 March 1982) of continuous observations which includes all sky camera photography every 30 min during daylight. Various cloudiness categories were identified and the radiation data averaged for 10 min centered on the time of the photograph. The global transmission ($K_t$) and diffuse fraction ($K_d$) were computed using eqns (4) and (5). The ensemble of these data in each cloudiness category were grouped within 5° ranges of elevation angle. Higher-order moments were also calculated. The mean values of $K_t$ vs zenith angle for various cloud conditions could then be calculated as well as $K_d$ and their standard deviations for the 13 mo period.

The observed averaged clear sky global transmittance vs elevation angle is shown in Fig. 1(a) for Davis. Also shown is the standard deviation (brackets denote ±1σ($K_t$)). Figure 1(b) shows $K_d$ for the same data set plotted vs $K_t$ along with the envelope defined by ±1σ($K_d$). The minimum number of observations is 12 at 70–75° with the typical number being 55 per elevation interval.

The observed variation of $K_t$ shows a strong dependence on solar elevation for elevations less than about 30°. The large standard deviations at low sun angles (<10°) is largely due to instrument errors and internal reflections. At higher angles, the large variance in the data most likely reflects the large variance in turbidity and precipitable water during the 13 mo observational period. Since the highest elevation angles are only attainable in summer and since the ambient conditions are least variable in summer and the extinction due to the various constituents is minimum at high elevation angles, the observed variance decreases significantly at solar elevation angles greater than 60°.

The diffuse fraction as a function of global transmissivity shows a clustering of the mean value about a straight line, but the variance at low values of $K_t$ (i.e. low elevation angle) is very large, again in part due to instrument errors and shading disk positioning errors at low sun elevation angles. What the two plots show is that in an average sense, $K_t$ is a well-behaved function of elevation angle (i.e. of air mass for fixed turbidity and water vapor content) and that the diffuse fraction in a mean sense is a well-behaved function of $K_t$. This dependence is well described by a linear regression equation

$$K_d = 0.88 - 1.024K_t$$

for which $R = 0.98$ and $\sigma$ for the fit is 0.022. The next question is, how well can the various parameterized transmission models reproduce these mean variations of $K_t$ and $K_d$.

The models were all run for the eleven sets of precipitable water and turbidity shown in Table 1. The average $K_t$ and $K_d$ and their standard deviations were calculated as a function of sun elevation for these 11 conditions and plotted in a format similar to Fig. 1.

Figure 2 contains the results for the RSC model. Figure 2(a) shows $K_t$ vs solar elevation with the brackets on the model predictions representing ±1σ variations in the model predictions for the range of $B_t$ and $P_w$ used in the calculations. The lines represent the mean observed values of $K_t$ and the ±1σ envelope. The model predicted values of
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Fig. 2. Plot of $K_t$ vs elevation (a) and $K_d$ vs $K_t$ as calculated by RSC model for clear sky. Precipitable water and turbidity coefficients were varied (cf. Table 1) with the average $\beta = 0.14$ and average $\gamma = 0.65$. Solid and dashed lines signify the mean value and 1σ variations in the observation plotted Fig. 1. Vertical brackets are ±1σ variations in the model predictions due to varying $\beta$ and $\gamma$.

$K_d$ are shown in Fig. 2(b) with the brackets illustrating the range of $K_d$ for the range of $\beta$ and $P_w$ used. The observed values and the ±1σ variations in the observed diffuse fraction are also shown. For sun elevations above 30°, the model does a fairly good job in predicting $K_t$, but generally underestimates the actual global radiation below 40° elevation. The predicted diffuse fraction is quite good, but the actual diffuse amount is underestimated just as the global radiation is underestimated for elevation angles less than 40°.

The MAC model predictions are shown in Fig. 3, in the same format as in Fig. 2. Clearly, this model is quite good at elevation angles ≤30° but overestimates the global radiation at elevations >40°. The diffuse fraction dependence on $K_t$ has the right slope but is systematically higher. Since $K_t$ is also overestimated, at $K_t \geq 0.7$ the actual diffuse component is significantly overestimated by this model.

Fig. 3. Same as Fig. 2 but for the MAC model.

Figure 4 shows the DNS model comparison with observations. Clearly it overestimates the global radiation at all sun angles and the diffuse radiation is greatly overestimated.

Figure 5 shows the comparison with the ABG model. Since the model predicts only global radiation, only the $K_t$ vs elevation relation can be shown. This model most closely approaches the shape of the observed distribution of $K_t$ but systematically underestimates the global radiation by about 40% at $\alpha \approx 10°$ and by about 7% at $\alpha \approx 60°$.

In Fig. 6, the normal incidence direct radiation as measured and as predicted by the RSC and MAC models is shown. It is clear that both models underestimate transmission for large air masses and overestimate the direct transmission at low air mass as compared with the observations. Note, however, that the observations taken over 13 mos contain some implicit seasonal bias. That is, the higher sun elevation angles are only available in summer when turbidity and total water vapor content can be high. The lower sun elevations are equally distributed among seasons except that the frequency of no cloud conditions is higher in summer and so there are more summer observations at all angles than
winter observations. Finally, in winter, clear (cloud free) days tend to be low turbidity days following cold front passage with lower precipitable water in the air mass. The model results as summarized, give equal weighting to all elevation angles and all tested conditions of turbidity and water vapor. If we chose to weigh the model calculation to use higher frequency of lower $\beta$ and $P_w$ at small elevations and higher $B_t$ and $P_w$ at large elevations the RSC model could be made to fit the observation to within $\pm 1\sigma(K_t)$ overall values of solar elevations. Similarly, by judicious choices of $B_t$ and $P_w$ at each value of sun elevation, the ABG model or the MAC model could be made to fit the observations exactly.

3. CLOUD EFFECTS

3.1 Model formulations

3.1.1 Cloud effects are the most difficult to model realistically because they have such a large variability. Clouds reflect, absorb, and scatter sunlight by amounts determined by the cloud particles’ phase and size and by the cloud thickness, altitude, and location relative to the sun. In general, clouds decrease global radiation except for cumulus clouds which can act to increase local global radiation at the ground.

Two cloud model types are in the literature, the first (e.g. Robinson) uses actual sunshine ($n$) vs potential sunshine ($N$) as a measure of cloudiness so that the cloudy sky global radiation is reduced to $n/N$ of its clear sky value, i.e.

$$G_{\text{cloud}} = G_{\text{clear}} \cdot \frac{n}{N} + I_c f(n/N).$$

(31)

The second term is the cloud produced diffuse radiation. The function $f(n/N)$ is given as

$$f(n/N) = 0.4 - 0.4(n/N)^2.$$  

(32)

Since this formulation is independent of cloud type and requires the actual “sunshine” ($n$) to be known, it is of limited usefulness. Replacing $n$ with (1 -
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where $C$ is fractional cloud cover is not appropriate.

A method more amenable to using cloud data as recorded in weather observations is that of Haurwitz[12] in which cloud transmission is defined by

$$T_{ci} = 1 - (1 - t_i)C_i,$$  \hspace{1cm} (33)

where $t_i$ is a function of cloud type and path length and $C_i$ is the fractional amount of that cloud in the $i$th cloud layer. The global radiation with cloud is then

$$G_{o} = G \cdot \sum_{i=1}^{l} T_{ci}.$$  \hspace{1cm} (34)

Models differ in how $t_i$ is defined. Both the DSN and MAC models use a Haurwitz formulation. In the MAC model this takes the form of

$$t_i = a' e^{-b'm},$$  \hspace{1cm} (35)

where $a'$ and $b'$ are specified for each cloud type. The DSN model uses a variation:

$$t_i = \frac{a}{Gm} l^{-b' m},$$  \hspace{1cm} (36)

in which $G$, the clear sky global radiation, must be in kWh/(m$^2$ day).

The ABG model uses a linear relationship:

$$t_i = X + Ym$$  \hspace{1cm} (37)

again with $X$ and $Y$ being defined for each cloud type. Values of the cloud specific coefficients are listed in Table 2.

In addition, further variations on the treatment of clouds include interlayer reflections[4] and other cloud transmission formulations (cf. review by Atwater and Ball[6]). These variants were not tested here either because previously reported tests show them to be less representative than those contained herein or in the case of reflection effects, the overall crudeness of the basic transmission models would not seem to warrant these additional refinements.

3.1.2 Observations. The observed dependence of $K_t$ on sun elevation for all cloud types in three ranges of cloud amount is shown in Fig. 7. The number of observations of each cloud type for each cloud amount is listed in Table 3. Clearly, for the measurement site, the vast majority of clouds occurring in amounts less than 0.66 are high clouds so the $K_t$ dependence shown in Fig. 7 is for cloudiness made up primarily of high cloud. For these clouds, $K_t$ is less than the clear sky value for 50° sun elevation or less and is greater at very high sun elevations.

In Fig. 8, the values of $K_t$ vs cloud type are shown for cloud amounts greater than 0.66. Clearly the transmission increases with cloud altitude. With near overcast of high cloud the global transmission is nearly that of clear sky, although the partition between diffuse and direct will differ. There is considerable scatter in the data as expected from the variability of the cloud large scale morphology and microstructure.

For scattered cloud conditions ($N \leq 0.33$), there is no obvious dependence of $K_t$ on cloud altitude (Fig. 9). Even though the sample population for low clouds is quite small, the very different effects of low cumuliform and low stratiform clouds is quite evident. The data for intermediate cloud cover shows even less dependence on cloud type. The dependence of diffuse fraction on global transmission for each cloud amount category is illustrated

Table 2. Coefficients for various models of cloud transmittance

<table>
<thead>
<tr>
<th>Model:</th>
<th>DSN</th>
<th>ABG</th>
<th>MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient:</td>
<td>$a$</td>
<td>$b$</td>
<td>$X$</td>
</tr>
<tr>
<td>Low cu-form</td>
<td>0.0404</td>
<td>0.104</td>
<td>0.366</td>
</tr>
<tr>
<td>Middle cu-form</td>
<td>0.0611</td>
<td>0.112</td>
<td>0.546</td>
</tr>
<tr>
<td>Hi cu-form</td>
<td>0.0956</td>
<td>0.079</td>
<td>(0.905)</td>
</tr>
<tr>
<td>Low st-form</td>
<td>0.0277</td>
<td>0.159</td>
<td>0.268</td>
</tr>
<tr>
<td>Middle st-form</td>
<td>0.0454</td>
<td>0.063</td>
<td>0.413</td>
</tr>
<tr>
<td>Hi st-form</td>
<td>0.1013</td>
<td>0.148</td>
<td>0.905</td>
</tr>
</tbody>
</table>
Table 3. Number of observations of cloud type by ranges of cloud cover

<table>
<thead>
<tr>
<th>Type</th>
<th>$0 &lt; N \leq 0.33$</th>
<th>$0.34 \leq N \leq 0.66$</th>
<th>$0.67 \leq N \leq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>15</td>
<td>5</td>
<td>171</td>
</tr>
<tr>
<td>Middle</td>
<td>51</td>
<td>29</td>
<td>382</td>
</tr>
<tr>
<td>High</td>
<td>212</td>
<td>134</td>
<td>174</td>
</tr>
</tbody>
</table>

in Figs. 10–12. As can be seen, $K_d$ variations for single cloud decks and for multiple cloud decks appear to be linearly dependent on $K_t$, independent of cloud types, i.e.

$$K_d = A + BK_t.$$  (38)

Table 4 lists the values of $A$ and $B$ for the various categories of cloudiness along with the regression fit coefficient $R$, and standard deviation $\sigma$, of the observations from the fitted line. The last entry represents coefficients applicable to any cloud condition. What these data show is that attempts to predict a single event, i.e. given time of day, cloudiness data, and an estimate or measure of $K_t$, the probability of predicting the instantaneous diffuse radiation to within 20% is low.

In Fig. 15 the regression equations for those cloud amounts for which $R > 0.8$ are plotted. Also shown are the standard deviations of the observations from the regression line. The extent of the lines denotes the range over which observations actually were available. At all values of $K_t$, the difference in diffuse fraction with and without clouds is quite apparent and significant and is the order of 0.2.

Given the scatter in the cloudy sky observations of $K_d$ vs $K_t$, the differences among the regression lines for differing cloud conditions is of marginal significance at best. Averaging coefficients for lines c, d, and f gives the values of the last line of Table 4.

These agree moderately well with the coefficients suggested by Page[13] ($A = 1.0, B = -1.096$) and in fact if the clear-sky data were averaged in with cloudy data (i.e. combining lines a and h, the coefficients match to three significant figures. The coefficients suggested by Spencer[14] ($A = 1.35, B = -1.67$) do not fit either the cloud or clear data well at all.

3.1.3 Model comparisons for $K_t$. The global transmittance predicted by the MAC and ABG cloud models are shown in Fig. 13, in a format similar to Fig. 7, using the average predicted values weighted by the frequency of occurrence of cloud types in each cloud amount category listed in Table 3. As with the observations, both models predict very little difference in transmission with cloud amount. For sun elevation angles less than 40°, both models predict the observed $K_t$ values quite well for all cloud amounts. At higher sun angles, the models significantly underestimate the transmission. This may be due to the climatic differences between the data sets used to determine the cloud transmission coefficients listed in Table 2. These data were predominantly mid-latitude, continental locations or east coast locations whereas the Davis site is characterized by a Mediterranean climate. The significant point is that the continental climates
Fig. 10. (a) Observed diffuse fraction vs $K_t$ for scattered cloud conditions ($N \leq 0.33$) for single cloud layer conditions (squares = high, triangles = middle, circles = low cloud). Line represents regression fit with coefficients given in Table 4. (b) Same as (a) except for multiple cloud layers (open circles = middle and high, triangles = low and high and closed circles = low and high).

Table 4. Values of regression line coefficients and fit parameters for various cloud categories for predicting $K_d = A + B K_t$.

<table>
<thead>
<tr>
<th>Cloud condition</th>
<th>Fig. 15. symbol</th>
<th>$A$</th>
<th>$B$</th>
<th>$R$</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cloud</td>
<td>a</td>
<td>0.88</td>
<td>-1.024</td>
<td>0.98</td>
<td>0.02</td>
</tr>
<tr>
<td>$N \leq 0.33$ single deck</td>
<td>b</td>
<td>0.981</td>
<td>-1.047</td>
<td>0.68</td>
<td>0.13</td>
</tr>
<tr>
<td>$N \leq 0.33$ multiple decks</td>
<td>c</td>
<td>1.079</td>
<td>-1.045</td>
<td>0.83</td>
<td>0.11</td>
</tr>
<tr>
<td>$0.34 \leq N \leq 0.66$ single deck</td>
<td>d</td>
<td>1.190</td>
<td>-1.248</td>
<td>0.84</td>
<td>0.09</td>
</tr>
<tr>
<td>$0.34 \leq N \leq 0.66$ multiple decks</td>
<td>e</td>
<td>1.112</td>
<td>-0.929</td>
<td>0.59</td>
<td>0.15</td>
</tr>
<tr>
<td>$0.66 &lt; N$ single deck</td>
<td>f</td>
<td>1.216</td>
<td>-1.184</td>
<td>0.91</td>
<td>0.07</td>
</tr>
<tr>
<td>$0.66 &lt; N$ multiple decks</td>
<td>g</td>
<td>1.070</td>
<td>-0.696</td>
<td>0.33</td>
<td>0.16</td>
</tr>
<tr>
<td>Average for all cloud conditions</td>
<td>h</td>
<td>1.11</td>
<td>-1.16</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>
Fig. 11. Same as Fig. 10, but for \(0.33 < N \leq 0.66\).
Fig. 12. Same as Fig. 10, but for $N > 0.66$. 
can have thick cloud decks and rain events in any season and cloud optical thickness in summer can be just as large as in winter. In a Mediterranean climate, summer cloudiness (i.e. high sun elevation periods) is infrequent and cloud thicknesses are typically very thin. Recognizing this climatological bias, it appears that the MAC and ABG models are reasonably accurate predictors of the dependence of $K_t$ on sun elevation for temperate continental and maritime climates. The Davis data are best interpreted as valid in Mediterranean climates with the major difference that cloud transmission at sun elevations >40° or 50° are higher in such climates than in more classical temperate continental or maritime climates.

Comparing model outputs for cloud amounts equal to 0.84 (Fig. 14) with the data in Fig. 8, shows that the MAC model does a better job of mimicking the distribution by cloud type but tends to underestimate the transmission of high cloud, overestimate the transmission of low cloud but does a very good job with the middle clouds. It should be remembered, however, that with nearly full overcast of low clouds, the presence of higher cloud layers is not detectable but their presence is likely in many situations. In such situations single cloud layer calculations will overestimate atmospheric transmission. Again, it should also be noted that these differences may be entirely due to differences in the nature of clouds between climatological zones.

4. SUMMARY AND CONCLUSIONS

For clear sky conditions and in the absence of measured global radiation, the bulk transmission models of Davies and McKay[5] and Atwater and Ball[6] give reasonable estimates of the global radiation for appropriate estimates of turbidity and precipitable water. However, the MAC model is biased toward overestimates of the direct radiation and the diffuse fraction. The global radiation so calculated should not be viewed as that which would be measured at an instant at a point but represents the most probable value given that the ambient conditions are correctly specified. For modeling pur-
poses, these predictions can be considered representative instantaneous values.

For cloudy conditions, the cloud transmission models of Haurwitz[12] and of Atwater and Ball[6] agree well with observations at Davis except at high sun elevations. This difference is attributed to the significantly different frequency and characteristics of summer clouds in a Mediterranean climate vs a temperate or tropical climate. For climates like that of California, one should increase the cloudy sky transmission predicted by the MAC (Haurwitz) model by about 15-20% for cloud cover ≥0.66. For near overcast conditions, the MAC underpredicts high cloud transmission by up to 10%, does a good job with middle cloud, and overestimates low cloud global transmission by 50%.

Clearly, the diffuse fraction variation with global transmission for clear skies is very different from cloudy sky conditions. One would expect that extinction due to suspended particles and atmospheric gases and the portion of scattered radiation directed downward relative to the total extinction should be different in clear skies than in the presence of clouds. Each cloud would be expected to produce a unique ratio of diffuse to global radiation. Although there is a great variety in large scale cloud parameters and in the microscale characteristics of the cloud particles, a single average relationship for all cloud conditions appears to be sufficient to define a representative value of diffuse fraction for a given set of conditions.

Given an estimate (model calculation or measurement) of \( K_t \), the diffuse fraction can be estimated for cloud-free conditions as

\[
K_d = 0.88 - 1.024K_t
\]

(39)

and for cloudy conditions

\[
K_d = \min \left\{ 1.11 - 1.16K_t, 1.0 \right\}
\]

(40)

With the \( K_t \) prediction models recommended here and eqns (39) and (40), the routinely recorded meteorological data variables relative humidity, cloud amount, cloud type, and visibility (to estimate turbidity) are sufficient to estimate, with confidence, a representative value of the global and diffuse radiation on a horizontal surface.

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**NOMENCLATURE**

- \( a, a' \) coefficients in Beer's law form of the cloud transmission parameter \( t \) for DSN, MAC models
- \( b, b' \) exponents coefficient in the DSN, MAC model for the cloud transmission parameter \( t \)
- \( A, B \) intercept value and slope of linear dependence of \( K_d \) on \( K_t \)
- \( C \) fractional cloud cover
- \( e_0 \) partial pressure of water vapor at the surface
- \( I_t \) diffuse irradiance on horizontal surface
- \( I_r \) direct irradiance on horizontal surface
- \( I_{w1}, I_{w1} \) direct irradiance on horizontal surface in the absence of any scattering, Rayleigh scattering, Mie scattering, respectively
- \( f_1, f_2 \) forward scattering fraction for Rayleigh, Mie scattering
- \( f_3 \) correction to diffuse irradiance due to surface albedo not equal to 0.25
- \( f \) cloud diffuse irradiance function
- \( G \) global irradiance on horizontal surface
- \( K_d \) fraction of diffuse irradiance in the global radiation, \( K_d = I_d/I \)
- \( K_t \) global transmission (clearness index, cloudiness index); ratio of global radiation at the ground to the polar irradiance at the top of the atmosphere: \( K_t = G/I_{(top)} \)
- \( k \) particle transmission parameter [eqn (18)]
- \( m_t, m \) air mass (path length) for mean sea level and actual station elevation
- \( p, p_0 \) atmospheric pressure at station, at mean sea level
- \( R \) goodness of fit parameter for least squares regression
- \( r_a, r_b \) reflection coefficients for ground, atmosphere
- \( S, S' \) average and actual (distance corrected) value of the solar constant. \( S = 1371 \text{ W/m}^2 \)
- \( t \) cloud type transmission parameter
- \( T_a \) transmission of the atmosphere after extinction due to process denoted by the subscripts. The subscripts are
  - \( a \) absorption by fixed gases
  - \( c \) extinction by clouds
  - \( p \) Mie scattering by particles
  - \( R_1 \) Rayleigh scattering by fixed gases
  - \( R_2 \) Rayleigh scattering by variable gases
T_a absorption by water vapor
T_g net global transmission which is equal to the product of the process transmissivities
U_o total column amount of ozone
X intercept in the MAC model formulation for t
\times total ozone path length in DSN model
Y slope of dependence of \tau on m in the MAC model
y precipitable water in cm
Z zenith angle of the sun
\beta angstrom turbidity coefficient
\chi total ozone in MAC model
\gamma Rayleigh scattering extinction coefficient
\sigma(q) standard deviation of quantity q
\tau, \tau_d air temperature, dew point temperature

REFERENCES