A model to predict expected mean and stochastic hourly global solar radiation $I(h;n_j)$ values

S. Kaplanis*, E. Kaplani
Renewable Energy Laboratory, T.E.I. of Patra, Meg. Alexandrou 1, Patra 26334, Greece

Received 26 July 2005; accepted 27 June 2006
Available online 20 September 2006

Abstract

This paper describes an improved approach for (a) the estimation of the mean expected hourly global solar radiation $I(h;n_j)$, for any hour $h$ of a day $n_j$ of the year, at any site, and (b) the estimation of stochastically fluctuating $I(h;n_j)$ values, based on only one morning measurement of a day. Predicted mean expected values are compared, on one hand with recorded data for the period of 1995–2000 and, on the other, with results obtained by the METEONORM package, for the region of Patra, Greece. The stochastically predicted values for the 17th January and 17th July are compared with the recorded data and the corresponding values predicted by the METEONORM package. The proposed model provides $I(h;n_j)$ predictions very close to the measurements and offers itself as a promising tool both for the on-line daily management of solar power sources and loads, and for a cost effective PV sizing approach.

© 2006 Elsevier Ltd. All rights reserved.

Keywords: Hourly solar radiation; Prediction; On-line management; Statistical simulation; Dynamic PV-sizing

1. Introduction

The prediction of the global solar radiation, $I(h;n_j)$, on an hourly, $h$, basis, for any day, $n_j$, was the target of many attempts [1–13]. Review papers, such as [1,2], outline the methodologies to obtain mean expected $I(h;n_j)$ values. A reliable methodology to predict $I(h;n_j)$, based on the least available data, taking into account morning measurement(s) and...
simulating the statistical nature of $I(h; nj)$, is a challenge. A model to predict $I(h; nj)$ close to real values would be useful in problems such as

1. effective and reliable sizing of the solar power systems i.e. PV generators [14].
2. management of solar energy sources; e.g. output of the PV systems, as affected by the meteo conditions, in relation to the power loads to be met.

The above issues drive the research activities towards the development of an improved effective methodology to predict the $I(h; nj)$, for any day $nj$ of the year at any site with latitude $\phi$, for a dynamic sizing of solar energy systems. One of those methodologies to predict the mean expected hourly global solar radiation, as proposed by the authors [15], provided a simple approach model based on the function

$$I(h; nj) = \alpha + \beta \cos(2\pi h/24),$$

where, $\alpha$ and $\beta$ are constants, which depend on the day $nj$ and the site $\phi$.

Eq. (1) shows some similarities to the Collares–Pereira and Rabl model [11]. However, both models differ in the way $\alpha$ and $\beta$ parameters are estimated. The $I(h; nj)$ model [15], as studied in detail, overestimates somehow $I(h; nj)$ at the early morning and late afternoon hours, while it underestimates $I(h; nj)$ around the solar noon hours. Although, the predicted
$(h; n_j)$ values fall, in general, within the range of the standard deviation of the measured, $I_{mes}(h; n_j)$ fluctuations, a more accurate and dynamic model had to be developed, for the needs of the on-line system management and cost-effective PV-sizing. Such a model should have inbuilt statistical fluctuations, as is the case with the METEONORM package [13], but with a more effective prediction power. A comparison of the predicted results between the METEONORM approach and the one to be proposed in this paper, as compared to measured $I_{mes}(h; n_j)$ values, during the period 1995–2000 for the region of Patra, Greece, will be presented, hereafter.

2. Model description

Due to the above-mentioned drawback of the simple model of Eq. (1), a correction factor is introduced, which takes into account the difference in the air mass, that the global solar radiation penetrates during the daytime hours. This new factor, normalized at solar noon, takes the form

$$e^{-\mu(n_j)\times\theta_z}/e^{-\mu(n_j)\times\theta_z;\omega=0},$$

where $\theta_z$ is the zenith angle and $\mu(n_j)$ the solar beam attenuation coefficient, determined in this approach by

$$\mu(n_j) = -\ln\left(\frac{H(n_j)}{H_{ext}(n_j)}\right)/x_m.$$  

$H_{ext}$ is the extra-terrestrial radiation during the day $n_j$ [18]. $H(n_j)$ is the daily global solar radiation at horizontal. For the case of Greece, it was obtained from a database of monthly radiation data [16]. For instance, the 12 monthly global solar energy, $E(n_j)$, values, given in the databank for the 6 climatic zones of Greece; see Fig. 1, are fitted in a function, as in Eq. (4), below. The fitting results provide the daily global solar radiation

![Fig. 1. The six climatic zones of Greece based on the mean monthly solar radiation.](image-url)
where,

\[ H(n_j) = C_1 + C_2 \cos\left(\frac{2\pi}{364}n_j + C_3\right), \]  

(4)

\( C_1 - C_3 \) are given in Table 1, for the 6 climatic zones of Greece. Note that, the correlation coefficient for all cases is higher than 0.996. Corresponding values for any region may be obtained the same way.

Furthermore, \( x(\theta_z) \) is the distance the solar beam travels within the atmosphere and \( x(\theta_z; \omega = 0) \) is this distance at solar noon (\( \omega = 0 \)). Notice that

\[ x(\theta_z) = -R_g \cos(\theta_z) + \sqrt{R_g^2 \cos^2(\theta_z) + (R^2 - R_g^2)} \]  

(5)

and

\[ R = R_g + H_{atm}, \]  

(6)

where \( R_g \) is the earth’s radius, \( R_g = 6.35 \times 10^3 \) km, and \( H_{atm} \) is the height of the atmosphere, \( H_{atm} = 2.5 \) km for the calculations. The results provided by the program did not change essentially with changing \( H_{atm} \) values. Also, \( \cos(\theta_z) \) is given by Eq. (7), where \( \delta \) is the solar declination and \( \omega \) the hour angle.

\[ \cos(\theta_z) = \cos(\phi) \cos(\delta) \cos(\omega) + \sin(\phi) \sin(\delta). \]  

(7)

For \( \omega = 0 \) or equivalently \( h = 12 \), i.e. at solar noon, Eq. (7) gives

\[ \cos(\theta_z;0) = \cos(\phi) \cos(\delta) + \sin(\phi) \sin(\delta) = \cos(\phi - \delta) \]  

(8)

or

\[ \theta_z;0 = \phi - \delta. \]  

(9)

Notice that, at sunset \( \theta_z = 90^\circ \). Therefore, Eq. (5) gives for sunset

\[ x(\theta_z = 90^\circ) = x(\omega_{sa}) = \sqrt{R^2 - R_g^2}. \]  

(10)

\( x_m \) is the mean daily distance the solar beam travels in the atmosphere. It is determined by the formula

\[ x_m = \frac{\sum_{i=1}^{n} x(\theta_z)_i}{N}. \]  

(11)

\( i \) is associated to the hour interval \( h \); also, \( \theta_z \) and, therefore, \( x(\theta_z) \) are determined by Eqs. (7) and (5) respectively.
For more accurate predictions, $x_m$ should be weighted over the hourly global solar intensity. That provides for $x_m$ the following formula:

$$x_m = \frac{\sum_{i=1}^{n} (x(\theta_i)I(h; n_j))}{\sum_{i=1}^{n} I(h; n_j)}.$$  \hfill (12)

This $x_m$ notion is in favour of winter times.

Finally, the proposed model, which gives better $I(h; n_j)$ prediction results than Eq. (1), for the mean expected hourly global solar radiation, takes the form

$$I(h; n_j) = A + B e^{-\mu(n_j)x(h)} \cos(\frac{2\pi h}{24}) e^{-\mu(n_j)\pi(h=12)}.$$  \hfill (13)

$A$ and $B$, in Eq. (13), are determined the same way as in [15], using the boundary conditions highlighted below. However, the $A$ and $B$ values of Eq. (13), do not take the same values as $a$ and $b$ obtained by the model of Eq. (1).

The two boundary conditions are

1. $I(h; n_j) = 0$ at $h = h_s$
   
   with $h_{ss} = 12 + \omega_{ss}/15^\circ$ and $\omega_{ss} = a \cos(-\tan(\varphi) \tan(\delta)).$  \hfill (14)

2. Integration of Eq. (13) over a day, from $\omega_{sr}$ to $\omega_{ss}$, provides $H(n_j)$ at the left side of Eq. (13). Hence, it gives

$$H(n_j) = 2A \int_0^{\omega_{ss}} d\omega + 2B \int_0^{\omega_{ss}} e^{-\mu(n_j)x(\theta)} e^{-\mu(n_j)\pi(h=12)} \cos(\frac{2\pi h}{24}) dh.$$  \hfill (15)

$x(\theta_i)$ is determined by Eqs. (5) and (7). From Eqs. (14) and (15), one may obtain $A$ and $B$, which are $n_j$ and $\varphi$ dependent.

3. Comparison of the mean expected values, predicted by this model, with the measured values

Predictions of mean expected $I_{m,pr}(h; n_j)$ values by this model, (see Eq. (13)), were compared against the measured $I_{mes}(h; n_j)$ values for Patra, Greece (where $\varphi = 38.25^\circ$ and $L = 21.73^\circ$). The $I_{mes}(h; n_j)$ values were recorded during the period 1995–2000. The predicted mean values, $I_{m,pr}(h; n_j)$, are shown in Figs. 2 and 3 for the 17th January and 17th July, for Patra, Greece, along with $I_{mes}(h; n_j)$.

For the validation of the results of this method, several dates were selected along the year to compare the predicted values and validate the model. The recorded $I_{mes}(h; n_j)$ values show statistical fluctuations, whose means are fairly close to the predicted values, both for January and July (see Figs. 2 and 3). On the other hand, for winter months, e.g. January, fluctuations are really strong. Therefore, prediction of statistically fluctuating $I(h; n_j)$ values, which may simulate closely the measured data, is a requirement both for the sizing projects and the on-line management.

The measured data for Patra city in Greece were tested with the Shapiro–Wilk statistic for normal distribution. In almost all cases, 19 out of the 20, the $I_{mes}(h; n_j)$ values satisfied the above test for normal distribution. For summer months, the standard
deviation of the hourly means of solar radiation, take values which for morning and late afternoon hours lie around $\sigma/I = 5$–8%, while for hours around the solar noon $\sigma/I$ lies within the range of 2–3%. On the contrary, for winter months $\sigma/I$ is around 25% for morning and late afternoon periods, while $\sigma/I$ is about 12% around the solar noon.

Figs. 4 and 6 show, respectively, the means of $I_{\text{mes}}(h; n_j)$, as determined from the measured data (1995–2000); the $I_{\text{m,pr}}(h; n_j)$ values, as predicted by the model developed in this research, and the $I_{\text{MET}}(h; n_j)$ values, provided by the METEONORM package, with the inbuilt random generator to provide for fluctuations, for the 17th January and 17th July. The METEONORM results show the presence of expected strong fluctuations in $I(h; n_j)$, especially, for winter months; see Fig. 4. Obviously, in summer period, fluctuations are neither strong nor frequent; see Fig. 6 for comparison. To smooth the $I_{\text{MET}}(h; n_j)$ values, one should take the hourly means for $\pm 2$ days around the 17th January, as seen in Fig. 5. The smoothed METEONORM predicted results are compared, in Fig. 5, with the mean predicted values by this method and the mean measured values for the 17th January for Patra, Greece.
Fig. 4. Values of the means for $I_{mes}(h;17)$, $I_{m,pr}(h;17)$ and $I_{MET}(h;17)$ for the 17th January, for Patra, Greece.

Fig. 5. A comparison between the predicted $I_{MET}(h;n)$ values by METEONORM for the days 15th–19th January and, consequently, their average values on one hand, and the predicted corresponding values $I_{m,pr}(h;n)$ by this model, with reference to the average measured $I_{mes}(h;n)$ values.

Fig. 6. Values of the means for $I_{mes}(h;198)$, $I_{m,pr}(h;198)$ and $I_{MET}(h;198)$ for the 17th July, for Patra, Greece.
4. Predicted $I(h;n_j)$ values by the stochastic approach of this model

Fluctuations of solar radiation data, like those recorded in Figs. 2 and 3, as well as those shown by the METEONORM results, in Fig. 5, affect the sizing process and the results obtained. However, the METEONORM package results:

(a) follow a pattern based on an inbuilt random generator which is not accessed.
(b) have the same profile for $I(h;n_j)$, without taking into account, as a reference, a morning $I(h;n_j)$ value. Therefore, the prediction is not really a dynamic one.

For an effective load management, on a daily basis, a more accurate and reliable prediction methodology for $I(h;n_j)$, with an inbuilt stochastic contribution for fluctuations, had to be established. The prediction methodology, proposed, takes into account a first early $I(h;n_j)$ measurement, at hour $h_1$. The model to predict $I(h;n_j)$ values for the remaining hours, as described in this paper, introduces a stochastic factor, which takes into account the previous hour $I(h-1;n_j)$ value. The steps followed to predict $I(h;n_j)$ based on a morning measurement, $I_{mes}(h;n_j)$, are outlined below.

1. Let $I_{m,pr}(h;n_j)$ predicted by Eq. (13). These values are easily generated with routines developed with MATLAB program for the purpose of this research project.
2. Let the solar intensity measurement at hour $h_1$ obtained be $I_{mes}(h_1;n_j)$ with a standard deviation $\sigma_f$.
3. $\sigma_f$ is pre-determined for the morning period $[h_s, h_s+s]$, afternoon period $(h_s-s, h_s]$, and hours around the solar noon $[h_s+s, h_s-s]$, with $s$ equal to 3 for winter and 4 for summer. Let the probability density function be a normal distribution with $(\sigma/I)$ 100% = 25% at morning and evening hours, and 12% around the solar noon, for winter months; and similarly 8% and 3%, respectively, for summer months. For example, at a morning hour $h$ and $n_j = 17$, $\sigma_f = 25%$ $I_{m,pr}(h;17)$.
4. Let us start with hour $h_1$. The program predicts $I_{m,pr}(h_1;17)$ and subtracts it from the measured $I_{mes}(h_1;17)$. The result is compared to $\sigma_f$, as in Eq. (16). Let this deviation $\delta I$ be $\lambda \sigma_f$.

$$\frac{I_{mes}(h_1;17) - I_{m,pr}(h_1;17)}{\sigma_f} = \frac{\delta I}{\sigma_f} = \lambda. \quad (16)$$

5. An attempt is made to predict $I_{pr}(h_2;n_j)$, at hour $h_2 = h_1 + 1$. The model gives an estimate of the $I_{pr}(h_2;n_j)$, taking into account the value of $\lambda$ determined in step 4. In this step, the model samples from a Gaussian probability density function in order to determine the $\lambda' \sigma_f'$ interval of the normal distribution in which the $I(h_2;n_j)$ value would lie. For instance, for $n_j = 17$ and a morning hour $h_2$, $\sigma_f' = 25% I_{m,pr}(h_2;17)$. $\lambda'$ is randomly selected through Gaussian sampling and is permitted to take, according to this model, values within the range $\lambda \pm 1$, with $\lambda$ now forced to an integer value. For values of $|\lambda| \geq 3$, the permitted range for $\lambda'$ would be either $[-4, -3]$ or $[3, 4]$ according to the sign of $\lambda$ for all day long. It is important to note that if $\lambda$ is as extreme as this, the $I_{pr}(h;n_j)$ lies in the same $\sigma_f$ region, for all day long, without jumping to other $\sigma_f$ intervals.
6. The predicted value at hour $h_2$, $I_{pr}(h_2;n_j)$, is determined based on the mean expected $I_{m,pr}(h_2;17)$ and the new deviation value $\lambda' \sigma_f'$, as in Eq. (17).

$$I_{pr}(h_2;n_j) = I_{m,pr}(h_2;n_j) \pm \lambda' \sigma_f'. \quad (17)$$
7. The model, then, compares the predicted $I_{pr}(h; nj)$ value with the mean expected $I_{m,pr}(h; nj)$. It repeats steps 4–6, for the hour $h_3$ and so on until the hour $h_{ss}/C0_1$.

Fig. 7 shows an example with the mean expected $I_{m,pr}(h; nj)$ values, as determined by the proposed model, see Eq. (13); the $I_{mes}(h; nj)$ values, as obtained from the measured data in 2000 for the 17th January ($nj = 17$), and the $I_{pr}(h; nj)$ values, as predicted by the proposed model with the inbuilt stochastic generator, taking into account an initial measured value $I_{mes}(8; 17)$ at 8 h, for the year 2000. The corresponding values for the 17th July ($nj = 198$), based on this model and triggered by an initial measurement taken at 7 h, are shown in Fig. 8.

The effect of the random sampling, i.e. the sequence of random numbers produced by the inbuilt random generator, on the predicted values $I_{pr}(h; nj)$, as estimated by the previously described stochastic model, is shown in Figs. 9 and 10. Predicted $I_{pr}(h; 17)$ values, for the whole day, starting with a measured value $I_{mes}(8; 17)$, as provided by the 2000 data, is displayed in Fig. 9. The corresponding graphs for the 17th July and $I_{mes}(7; 198)$, are shown in Fig. 10. Due to the small standard deviation values in summer months, the effect of this random value is expected to be very small. This may be realized through the closeness of the values in series $I_{pr} = 1$ to $-4$, in Fig. 10. A much larger effect is seen during the winter months (see curves in Fig. 9), which is, however, within the limits of one standard deviation.

Mean hourly predicted values $I_{m,pr}(hnj)$ and stochastically fluctuating ones, provided by this model based on only one-morning measurement, are shown against measured $I(h; nj)$ data for the years 1998–2000 in Fig. 11. It is evident from the comparison between predicted and real measured data that they lie very close to each other at the corresponding hours, especially for the years 1999 and 2000. For the year 1998 the prediction is not very good at noon hours due to an unexpected and really unusual large and rapid drop of the
Fig. 8. Mean predicted hourly global solar radiation values, $I_{m,pr}(h;198)$; the measured ones, $I_{mes}(h;198)$ for the 17th July 2000 and the predicted $I_{pr}(h;198)$ values, based on a single morning measurement at 7h, for Patra, Greece.

Fig. 9. Four runs (series Ipr−1 to −4) of the daily $I_{pr}(h;17)$ values based on the measured $I_{mes}(8;17)$ value for the 17th January and the 8h from the data records of 2000.

Fig. 10. Four runs (series Ipr−1 to −4) of the daily $I_{pr}(h;198)$ values based on the measured $I_{mes}(7;198)$ value for the 17th July and 7h from the data records of 2000.
I(h; nj) around the solar noon. Drops of this magnitude are not considered in this model which is built for rather mild changing weather conditions.

5. Conclusion

The proposed model predicts mean expected $I_{m,pr}(h; nj)$ values for any hour $h$ of a day $nj$, with a very good accuracy, as the comparisons with the measured $I_{mes}(h; nj)$ values, within a period of years (1995–2000), have shown. The predicted $I_{m,pr}(h; nj)$ values lie closer to the measured means, than the results obtained from Eq. (1) [15]. A program was developed in MATLAB to provide and plot the $I(h; nj)$ solar hourly estimations. The model was checked with repeated runs and different sequences of random numbers, as required for the prediction of $I(h; nj)$. The results were within the limits of the standard deviation and introduce no instability effect, for the daily on-line management and effective PV-sizing.

The dynamic part of the proposed model presents a very good behaviour, providing predicted values $I_{pr}(h; nj)$, based only on one morning measurement of $I(h_1; nj)$. The model permits $I(h; nj)$ to take stochastically based values inside the domain $I_{pr}(h; nj) \pm \sigma_1$ when passing from the $h_1$ interval to the $h_2$ one.

A similar behaviour, providing random fluctuations, is shown in the predicted results by METEONORM. However, its inbuilt dynamic predictive power provides the same $I(h; nj)$ profile and does not incorporate real daily variations, as is the case with the proposed model of this study, whose predictions are based on a real morning measurement, $I(h_1; nj)$.

It is important to note that examples of a worse case scenario given by the highly deviant data of year 2000 from the measured means of 1995–2000, were used to show the prediction power of this methodology.

The $I(h; nj)$ fluctuations do affect the sizing of PV generators to meet the loads and consequently, the load management based on a daily/hourly basis. The effect is really critical, if sizing is based on winter conditions and the PV system is a stand alone one [17].

The model proposed in this paper, which predicts global solar radiation during a day based on one morning measurement, has applications in predictive load management where the daily solar radiation profile needs to be determined in advance. The validity of the model has been examined for the climate of Greece and also holds for similar climates. Further work is currently being undertaken to gain further reliability in the proposed...
model when applied in northern climates, in which weather conditions change rapidly during the day. In this case the prediction of $I(h; n_j)$ takes into account two rather than one morning measurements, at hours $h_1$ and $h_2$, and the $I(h; n_j)$ rate of change over $h$ will be the prime criterion.

Acknowledgements

The project for the dynamic prediction of $I(h; n_j)$ was funded by the Greek Ministry of Education and specifically the Archimides programme. The data of the period 1995–2000 were provided by the Hellenic Meteo Organization (HMO).

References