Due Date: Friday February 27, 12 noon

The aim of this homework set is to help you become familiar with the physics of coupled harmonic oscillators.

1. Find the eigenfrequencies of a system of three equal masses, coupled between themselves and with the walls by springs with equal spring constants. How do these frequencies compare to the frequencies of the uncoupled system?

2. Three oscillators of equal mass are coupled in such a way that the potential energy of the system is given by

\[ U = \frac{1}{2} [k_1 (x_1^2 + x_3^2) + k_2 x_2^2 + k_3 (x_1 x_2 + x_2 x_3)] , \quad (1) \]

where \( k_3 = \sqrt{k_1 k_2} \). Find the eigenfrequencies of the system of oscillations. What is the physical interpretation of the zero-frequency mode?

3. Describe the motion of the system of oscillators characterized by the potential

\[ U = \frac{1}{2} k x_1^2 + \frac{1}{2} k x_2^2 + \frac{1}{2} k (x_1 x_2 + x_1 x_2 + x_1 x_2) \quad (2) \]

making the assumption that the displacements from the equilibrium configuration are small.

4. Find the eigenfrequencies of the system shown in the figure.