PHYS 528 — Spring 2004

Homework # 2

Due Feb 17

1. A magnetic system of $N$ spins is in thermodynamic equilibrium at temperature $T$. Let $\mu$ be the magnetic moment of each spin; and let $m$ be the total magnetic moment per spin, so $-\mu < m < \mu$. The alleged free energy per spin is

$$a(m, T) = \lambda(T) \left[ \left( \frac{m}{\mu} \right)^4 - \left( \frac{m}{\mu} \right)^2 \right] ,$$

(1)

where $\lambda(T) > 0$. Is this free energy acceptable? Why? Include a response function in your discussion.

2. A substance is found to have two phases, $N$ and $S$. In the normal phase $N$, the total magnetic moment $M$ is negligible. At a fixed temperature $T < T_c$, as the external magnetic field $B_0$ is lowered below the field

$$B_i(T) = B_i(0) \left[ 1 - \left( \frac{T}{T_c} \right)^2 \right] ,$$

(2)

the normal state undergoes a phase transition to a new phase $S$. In the superconducting phase $S$, the magnetic field inside the material $B = B_0 + \mu_0m$ —where $m$ is the total magnetic moment per particle (magnetization)— vanishes (“Meißner effect”).

(a) Find the difference in Gibbs free energies between the two phases at a temperature $T < T_c$, $G_S(T, B_0) - G_N(T, B_0)$.
(b) At $B_0 \leq B_i(0)$, compute the latent heat of transition $l$ from the $N$ to $S$ phase.
(c) At $B_0 = 0$, compute the discontinuity in the specific heat as the material transforms from the $N$ to $S$ phase. Is the phase transition first or second order at $B_0 = 0$?

3. Consider a closed box of fixed volume $V$ and internal energy $U$ containing black-body radiation and a black hole of mass $M$. The thermodynamic states of a black hole can be described by two EsOS (here $c = 1$):

$$U_h = M,$$

(3)

$$S_h = 4\pi M^2$$

(4)
(since the entropy $S_h = A/4$ in terms of the area of the black hole, $A = 16\pi M^2$). Use the corresponding equations for radiation (e.g., from problem 2 in HW #1), neglecting any changes due to the presence of the black hole.
(a) Find, for different values of $U$ and $V$, the values of $M$ so that radiation and hole are in equilibrium at a temperature $T$.
(b) Sketch the graph $T = T(U)$ for the system, also displaying lines of superheating and supercooling. Discuss the result.

4. For a van der Waals fluid,
(a) Calculate the Helmholtz free energy per mole, $A(T, V)/n = a(T, v = V/n)$.
(b) Expand the function
$$g(T, P; v) = a(T, v) + Pv$$
in powers of $\eta = v - v_c$, where $v_c$ is the critical volume per mole. Carry out the expansion to fourth order, and eliminate the cubic term by a shift in $\eta$. Find the coefficients of $\eta$, $\eta^2$, and $\eta^4$ in terms of $t = (T - T_c)/T_c$ and $p = (P - P_c)/P_c$, where $T_c$ ($P_c$) is the critical temperature (pressure).
(c) Expand the coefficients for small $t$ and $p$, and show they satisfy the behavior assumed in the Ginzburg-Landau theory.
(d) Sketch $g(T, P; v)$ as function of $\eta$ near (above and below) the critical point. Is $\eta$ a good order parameter? Why?

5. Calculate the critical exponents (degree of the critical isotherm $\delta$, degree of the coexistence curve $\beta$, exponents of heat capacity $\alpha, \alpha'$, and exponents of isothermal compressibility $\gamma, \gamma'$) of a van der Waals fluid.