Physics 141H
Solutions to Midterm #1
February 12, 2003

I: Multiple choice (4 pts. each)

For the multiple-choice questions, please provide a sentence or two explaining why you chose your answer. (A correct answer without explanation will receive 2 pts.)

1) A particle, which is constrained to move in one dimension, travels at constant velocity for 10s, then experiences a constant negative acceleration. Which of the following graphs best represents its position as a function of time?

For the first 10s, the distance will be given by \( x(t) = x_0 + vt \), a straight line. Afterwards, with constant negative acceleration \( a \), we will have \( x(t) = x_0 + v_0 t - \frac{1}{2}at^2 \), which is a parabola. Only curve (A) has both these features.
2) A particle of mass $m$ is released from a vertical height $h$ and horizontal distance $d$ on a curved ramp, as shown below.

Which of the following might be an expression for its velocity upon reaching the bottom (point $O$):

A) $v = \frac{h}{4m}g$

B) $v = \sqrt{2gh}$

C) $v = \frac{1}{2}mgh$

D) $v = h^2g$

Here we are looking for a velocity, and the choices given have the following units:

$$v = \frac{h}{4m}g = \frac{m \cdot m/s^2}{kg} = \frac{m^2}{s^2}$$

$$v = \sqrt{2gh} = \sqrt{m/s^2} \cdot m = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$

$$v = \frac{1}{2}mgh = kg \cdot m/s^2 \cdot m = \frac{kg \cdot m^2}{s^2}$$

$$v = h^2g = m^2 \cdot \frac{m}{s^2} = \frac{m^3}{s^2}$$

Only choice (B) has units of velocity, so it’s the only possible answer.
3) Which of the following statements can never be true:

A) A particle is moving at constant speed, and also accelerating
B) A particle is moving without accelerating, and its speed is decreasing
C) A particle has positive $x$ velocity, negative $x$ acceleration, and positive $x$ position
D) A particle, moving in one dimension, has positive acceleration and decreasing speed

Speed is the magnitude of the velocity vector, so it can only change if the velocity changes. Since a change in velocity is defined as acceleration, choice (B) is not possible.

4) Which of the following is not an example of Newton’s Third Law?

A) As you jump in the air, gravity from the Earth pulls you down, while gravity from your body exerts an equal force on the earth.
B) A horse pulls on a cart, and the cart pulls back equally hard on the horse.
C) A batter hits a baseball, propelling it forward, and the ball exerts an equal force backward on the bat.
D) A book rests on a table, with the force of its weight equal to the normal force from the table.

In example (D), the two forces mentioned act on the same object (the book). Therefore this cannot be an example of Newton’s Third Law, which concerns a pair of forces acting on different objects.
II. Problems:

Please show all your work (use the back of the paper if needed)

II.1 (25 pts):

A plane has a maximum airspeed of 250mi/hr. How long will it take to fly from city A to city B, 1000 mi apart, if there is a constant 50mi/hr wind blowing in the direction shown?

Answer:

I use the following coordinate system:

First, write the wind’s velocity vector in terms of its components:

The plane must direct its velocity vector to cancel the $v_y$ component of the wind’s velocity (since otherwise the plane would be blown off course and never arrive at B).

In other words, we want the plane’s velocity $v$ to have a $y$ component given by:

$$v_{y,\text{ground}} = v_{y,\text{air}} + w_y = 0$$

$$v_{y,\text{air}} = -w_y = -|w| \cos 45^\circ = -50\text{mi/hr} \cdot \cos 45^\circ = -35.3\text{mi/hr}$$
Since the plane’s maximum airspeed is 250 mi/hr, we therefore have:

\[ v_{\text{air}} = 250 \text{ mi/hr} = \sqrt{v_{x,\text{air}}^2 + v_{y,\text{air}}^2} \]

\[ v_{x,\text{air}} = \sqrt{(250 \text{ mi/hr})^2 - v_{y,\text{air}}^2} = \sqrt{(250 \text{ mi/hr})^2 - (35.3 \text{ mi/hr})^2} = 247 \text{ mi/hr} \]

Now we need to find the \( x \) velocity with respect to the ground:

\[ v_{x,\text{ground}} = v_{x,\text{air}} + w_x = 247 \text{ mi/hr} - |w| \cos 45^\circ = 247 \text{ mi/hr} - 35.3 \text{ mi/hr} = 212 \text{ mi/hr} \]

Therefore it takes \( t = \frac{d}{v} = \frac{1000 \text{ mi}}{212 \text{ mi/hr}} = 4.7 \text{ hrs.} \) to make the trip from A to B.
II.2 (25 pts): A plane flying at a constant altitude of 500m with a constant velocity of 100m/s wishes to drop an aid package to a specific point on the ground. From what horizontal distance $d$ should the package be dropped? At what angle does it hit the ground? You may assume that there is no wind, and that gravity is the only force acting on the package.

Answer:

The situation is as follows:

Since the plane is flying at constant altitude, it has no $y$ velocity when it is released, and its $x$ velocity is the same as the speed of the airplane. So the time it takes for the package to reach the ground can be found by:

$$ h = \frac{1}{2} g t^2 $$

$$ t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \cdot 500m}{9.8m/s^2}} = 10.1s $$

In that time it moves a horizontal distance of:

$$ d = v_x t = (100m/s)(10.1s) = 1.01km $$

To find the angle of impact, we need to know the $x$ and $y$ component’s of the package’s velocity at impact. The $x$ component is still 100m/s (constant throughout), and the $y$ component is given by (note that I’ve taken positive $y$ downward for this problem):

$$ v_y = gt = 9.8m/s^2 \cdot 10.1s = 99.0m/s $$
Therefore, $\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1} \frac{99 \text{m/s}}{100 \text{m/s}} = 44.7^\circ$

II.3 (25 pts.) A 10,000-kg spaceship is at rest in deep interstellar space, when the captain orders two of its thrusters to be fired, as shown below. These thrusters don’t turn on instantly, though, but rather supply a force that increases linearly with time as shown. Where will the spaceship be 100s after the thrusters begin firing?

Answer:

I use the following coordinate system:

The net force at time $t$ is given by:

$$F_x(t) = 1500 \frac{N}{s} t + 1000 \frac{N}{s} \cdot \cos 30^\circ \cdot t = 2370 \frac{N}{s} t$$

$$F_y(t) = 1000 \frac{N}{s} \cdot \sin 30^\circ \cdot t = 500 \frac{N}{s} \cdot t$$

Therefore, the acceleration is:
\[ a_x(t) = \frac{F_x(t)}{m} = \frac{2370 \frac{N}{s} \cdot t}{10000 \text{kg}} = 0.237 \frac{m}{s^3} \cdot t \]
\[ a_y(t) = \frac{F_y(t)}{m} = \frac{500 \frac{N}{s} \cdot t}{10000 \text{kg}} = 0.050 \frac{m}{s^3} \cdot t \]

We can now integrate to find the velocity:

\[ v_x(t) = \int a_x(t) \, dt = \int 0.237 \frac{m}{s^3} \cdot t \, dt = \frac{1}{2} \cdot 0.237 \frac{m}{s^3} \cdot t^2 = 0.119 \frac{m}{s^3} \cdot t^2 \]
\[ v_y(t) = \int a_y(t) \, dt = \int 0.050 \frac{m}{s^3} \cdot t \, dt = \frac{1}{2} \cdot 0.050 \frac{m}{s^3} \cdot t^2 = 0.025 \frac{m}{s^3} \cdot t^2 \]

Since the spaceship was at rest at \( t = 0 \), there are no \( v_o \)'s to worry about.

One more integration gives up the position:

\[ x(t) = \int v_x(t) \, dt = \int 0.119 \frac{m}{s^3} \cdot t^2 \, dt = \frac{1}{3} \cdot 0.119 \frac{m}{s^3} \cdot t^3 = 0.040 \frac{m}{s^3} \cdot t^3 \]
\[ y(t) = \int v_y(t) \, dt = \int 0.025 \frac{m}{s^3} \cdot t^2 \, dt = \frac{1}{3} \cdot 0.025 \frac{m}{s^3} \cdot t^3 = 0.0083 \frac{m}{s^3} \cdot t^3 \]

I’ve chosen the coordinates so that \( x_o = y_o = 0 \). Now we simply plug in \( t = 100 \text{s} \) to find:

\[ x(100 \text{s}) = 40 \text{km} \]
\[ y(100 \text{s}) = 8.3 \text{km} \]