Motion in One Dimension

• Much of the physics we’ll learn this semester will deal with the motion of objects
• We start with the simple case of one-dimensional motion
  – Or, motion in $x$:
  – As always, we begin by making observations. In this case we note the $x$ position of a particle at various times, e.g.:

<table>
<thead>
<tr>
<th>$t$(s)</th>
<th>$x$(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
</tr>
<tr>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>5</td>
<td>41</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
</tr>
</tbody>
</table>
• As we take more and more detailed measurements, (i.e., measure $x$ at more closely-spaced times), our table can get very long.

• More convenient to express the data in a graph:
Velocity

• We often want to know how fast (and in which direction) an object is moving

• This quantity is called velocity
  – Dimension is distance divided by time
  – Can be positive (x increasing) or negative (x decreasing)
  – Speed is the magnitude of the velocity (can’t be negative)

• Average velocity is defined as the net change in position divided by the elapsed time
  – Note that even though speed = |velocity|, average speed is not always equal to |average velocity|
  – Example: a car that drives 100 miles east at 55 mi/hr, then drives 100 miles west at the same speed, has average speed of 55 mi/hr…but average velocity of 0!
Average and Instantaneous Velocity

• Looking again at our example particle:
  - Travels from 0 to 30 meters in 6 seconds
    - Average velocity = $\frac{\Delta x}{\Delta t} = 5 \text{ m/s}$
  - But clearly there’s more going on – velocity is not constant
• Let’s say we want to know the velocity at \( t = 3.75 \) s
• We can zoom in the graph near this time:

Note that graph is nearly a straight line over this small \( \Delta t \)

- Velocity = \(-1.2 \text{ m} / 0.2 \text{ s} = -5.5 \text{ m/s} \) “near” \( t = 3.75 \) s
• To find velocity exactly at 3.75 s, need infinitely small \( \Delta t \)
The Derivative

• Infinitely small $\Delta$’s are known as *differentials* and written as $d$

• So, the velocity at time $t = \frac{dx}{dt}$

• On the graph, we can find this velocity by drawing a tangent line:
• But we’ll most often evaluate this mathematically
• Example:

\[ x(t) = 35 \text{ m} + (17 \text{ m/s})t + (8 \text{ m})\cos\left(\left(3s^{-1}\right)t\right) \]

• What is \( v \) at \( t = 4.2 \text{ s} \) ?

\[ v(t) = \frac{dx}{dt} = 17 \text{ m/s} - (24 \text{ m/s})\sin\left(\left(3s^{-1}\right)t\right) \]

• So \( v(4.2 \text{ s}) = 16.2 \text{ m/s} \)
  – Note that we measure angles in radians, unless otherwise specified
Acceleration

- Velocity measures how quickly (and in which direction) position is changing.
- *Acceleration* measures how quickly (and in which direction) velocity is changing.
- We can define average acceleration as $\Delta v/\Delta t$.
- But again, instantaneous acceleration is often the more interesting quantity:

$$a(t) = \frac{dv}{dt} = \frac{d(dx/dt)}{dt} = d^2x/dt^2$$

- From our previous example, $a(4.2 \, s)$ is given by

$$a(t) = \frac{dv}{dt} = -(72 \, m/s^2) \cos \left((3s^{-1})t \right)$$
• To summarize:
  \[ x(t) \]
  \[ \frac{dx}{dt} \]
  \[ v(t) \]
  \[ \frac{dv}{dt} \]
  \[ a(t) \]

• It turns out that the reverse process is even more useful…
Going from $a$ to $x$

- Since $x \to a$ meant taking derivatives, $a \to x$ means integrating.
- Let’s start with a simple case: $a(t) = A$

$$v(t) = \int a(t) \, dt = \int A \, dt = At + C_1$$

- By definition, $C_1 = v(0)$ is the initial velocity, $v_0$

$$x(t) = \int v(t) \, dt = \int [At + v_0] \, dt$$

$$= \frac{1}{2} At^2 + v_0 t + C_2$$

Or,

$$x(t) = \frac{1}{2} At^2 + v_0 t + x_0$$

Only true for constant acceleration!
Example: Objects in Free-fall

- Objects falling near the Earth’s surface have a nearly constant acceleration of 9.8 m/s$^2$ downward (independent of which object, if we ignore air resistance)
  - We call this value $g$
  - We’ll find out why it’s constant later in the course…

- So, we can apply the rules of constant-acceleration kinematics to understand their motion

- We already know enough the start solving “real” problems

- Sample problem:
  - Some of the best pitchers in baseball can throw a ball at about 100 mi/hr. How high would a ball thrown straight up with this height go?
• We know the acceleration is \( g \), so we should immediately think about using the equation relating position to acceleration:

\[
y(t) = \frac{1}{2} At^2 + v_0 t + y_0
\]

• \( A = -g \) in this case (we take upwards to be the positive direction)

• Life is easier if we set \( y_0 = 0 \), so we’ll do that (we can always define a convenient coordinate system)

• But we don’t know \( t \)...

• However, we’re interested in the \textit{maximum} height of the ball – in other words, it’s no longer climbing higher, but hasn’t started to fall back yet either. Therefore its velocity is 0
• To find the time when \( v = 0 \), we use the relation between velocity and acceleration:

\[
v(t) = v_o + At
\]

\[
v(t_{\text{max}}) = 0 = 100\text{mi/hr} - gt_{\text{max}}
\]

• Oops, we better convert everything to metric:

\[
\frac{100\text{mi}}{\text{hr}} \cdot \frac{5280\text{ft}}{1\text{mi}} \cdot \frac{12\text{in}}{1\text{ft}} \cdot \frac{0.0254\text{m}}{1\text{in}} \cdot \frac{1\text{hr}}{3600\text{s}} = 44.7\text{m/s}
\]

• Now we can find \( t_{\text{max}} \):

\[
t_{\text{max}} = \frac{44.7\text{m/s}}{9.8\text{m/s}^2} = 4.6\text{s}
\]
• Now we can find our answer:

\[ y(t) = \frac{1}{2} At^2 + v_0 t \]

\[ = \frac{1}{2} (-9.8 \text{m/s}^2)(4.6 \text{s})^2 + (44.7 \text{m/s})(4.6 \text{s}) \]

\[ = 102 \text{m} \]