Center-of-mass Energy

- When we first studied systems of particles, we learned that the center of mass was a very special position, for which Newton’s Laws could be applied as though the system were a single particle:

\[ \mathbf{F}_{\text{ext}} = m \mathbf{a}_{\text{cm}} \]

- Now let’s see what happens if the force is applied as the center of mass moves
- First, for an infinitesimally small distance:

\[ \mathbf{F}_{\text{ext}} \cdot d\mathbf{r}_{\text{cm}} = m \mathbf{a}_{\text{cm}} \cdot d\mathbf{r}_{\text{cm}} \]
• We can rewrite the right-hand side of the equation to find:

\[
F_{\text{ext}} \cdot dr_{\text{cm}} = m \frac{dv_{\text{cm}}}{dt} \cdot dr_{\text{cm}} = m \frac{dv_{\text{cm}}}{dt} \cdot \frac{dr_{\text{cm}}}{dt} = m v_{\text{cm}} \frac{dv_{\text{cm}}}{dt} dt = m v_{\text{cm}} dv_{\text{cm}}
\]

• Now we can integrate both sides of the equation:

\[
\int_{r_1}^{r_2} F_{\text{ext}} \cdot dr_{\text{cm}} = \int_{v_1}^{v_2} m v_{\text{cm}} dv_{\text{cm}} = \frac{1}{2} m \left[ v_2^2 - v_1^2 \right] = \Delta KE_{\text{cm}}
\]
• If the force happens to be constant, this simplifies to:

\[ \mathbf{F}_{\text{ext}} \cdot \int_{r_1}^{r_2} d\mathbf{r}_{\text{cm}} = \Delta KE_{\text{cm}} \]

\[ \mathbf{F}_{\text{ext}} \cdot \Delta \mathbf{r}_{\text{cm}} = \Delta KE_{\text{cm}} \]

• This equation looks a lot like the work-energy theorem
  – But it’s not!
  – The quantity on the left-hand side is not work
• For example, consider the case of a person jumping into the air
• The external force on the person before her feet leave the ground is \( N - mg \), so the quantity on the left would be

\[ (N - mg) \Delta y_{\text{cm}} \]
• But the normal force is applied at a constant position (the point of contact between feet and floor), and therefore it does no work

• The only external work is that due to gravity:

\[ W_{\text{ext}} = -mg \Delta y_{\text{cm}} \]
Example

- A 2000-kg car traveling at 50mi/hr runs into a concrete wall. The wall is very thick, and therefore moves negligibly in the collision. The front end of the car crumples, so the car’s center of mass moves forward 75cm during the collision. What is the force acting during the collision (if we assume it’s constant)?
- Note that conservation of energy isn’t much help here.
- Taking the car to be the system, we have:

\[ \Delta KE + \Delta U + \Delta E_{\text{int}} = W_{\text{ext}} \]

Because point where force is applied doesn’t move
• But we can use the center-of-mass motion:

\[ \mathbf{F}_{\text{ext}} \cdot \Delta \mathbf{r}_{\text{cm}} = \Delta KE \]

\[ -F_{\text{ext}} \Delta x_{\text{cm}} = -\frac{1}{2} m v_i^2 \]

\[ F_{\text{ext}} = \frac{1}{2} \frac{m v_i^2}{\Delta x_{\text{cm}}} = \frac{1}{2} \frac{2000\text{kg} \cdot \left( 50 \frac{\text{mi}}{\text{hr}} \cdot \frac{1609\text{m}}{\text{mi}} \cdot \frac{1\text{hr}}{3600\text{s}} \right)^2}{0.75\text{m}} \]

\[ = 670\text{kN} \]
Rotational Motion

- Many of the systems we encounter are made up of particles whose positions relative to each other are constant
  - These are “rigid objects”
  - Actually, no real object is perfectly rigid – the atoms in it can vibrate independently, for example. But the approximation of perfect rigidity is a good one for many solid objects

- We found early in the course that if such an object is moving in any direction, we can treat it as though it were a particle

- However, there is a type of motion that a rigid object can undergo that a particle cannot – rotational motion

- The study of rotational motion will be the next major topic in this course
Definition

• The first step in studying rotation is defining what it is.
• We say an object is rotation if we can define a line, or axis, such that:
  – Every part of the object undergoes circular motion, with the center of the circle on the axis, or
  – A perpendicular line drawn from the axis to any point in the object moves through the same angle in a given time interval.
Rotational Kinematics

• The first step in studying translational motion was to define the necessary kinematic quantities (position, velocity, acceleration), and their relation to one another.

• We’ll need to do the same for the variables that describe rotations
  – Though it’ll be easier this time since we can draw analogies to the linear kinematic quantities

• Consider an object rotating such that the axis of rotation is out of the page:
• At some later time, it looks like this:

![Diagram showing the object and point P]

• The point $P$ (and every other point in the object) has changed direction by an angle $\theta$

• Thus, $\theta$ is the rotational analog of distance
Some Notes on $\theta$

• We always measure $\theta$ in radians
  – This makes the relationship between angular and linear motion much clearer

• Just as position in one-dimensional linear motion can be positive or negative, so can $\theta$
  – By convention, we define the sign as follows:
    1. Call the axis of rotation the $z$ axis, and call the initial angle $\theta = 0$
    2. Look at the object from the positive $z$ axis
    3. If the object rotates counter-clockwise, $\theta$ becomes $> 0$
    4. If it rotates clockwise, $\theta$ becomes $< 0$

• There is no bound on the value of $\theta$
  – Absolute values greater than $2\pi$ indicate that the object has made more than one full rotation

• There is no dimension associated with $\theta$