Example 2: Non-equilibrium

• The 747 comes in for a landing at a speed of 70m/s. Before touching the ground, the wheels are not rotating. How long a skid mark do the wing wheels leave (assume their mass is 500kg which is distributed uniformly, radius is 1.3m, and the coefficient of friction with the ground is 0.5)?

• The tires will leave a skid mark if the speed of the tire in contact with the ground is less that the linear velocity of the plane
  – i.e., the ground is moving relative to the tire

• The condition for this is:

\[ \omega R < v \]

• So the tire will stop skidding (and start rolling) when it has attained an angular velocity of \( v/R \)
• The forces acting on the wheel after the plane touches down are:

• Since the wheel is not accelerating downward, we know that $N = W_w$
• The torque about the center of the wheel is:

$$\tau = fR = \mu W_w R$$
• The angular acceleration is:

\[
\alpha = \frac{\tau}{I} = \frac{\mu W_w R}{\frac{1}{2} m R^2} = \frac{2 \mu W_w}{m R}
\]

• So the time it takes for the wheel to stop skidding is given by:

\[
\omega = \omega_o + \alpha t = 0 + \alpha t = \frac{v}{R}
\]

\[
t = \frac{v}{R \alpha} = \frac{v}{R} \cdot \frac{m R}{2 \mu W_w} = \frac{mv}{2 \mu W_w}
\]

\[
= \frac{500 \text{kg} \cdot 70 \text{m/s}}{2 \cdot 0.5 \cdot 142 \text{kN}} = 0.25 \text{s}
\]
• This means that the skid mark has a length of:

\[ l = vt = 70 \text{m/s} \cdot 0.25 \text{s} = 17.5 \text{m} \]

• Note that this answer can’t be quite right
  – The same frictional force that causes the wheel to start spinning also slows down the plane
  – So we made the approximation that this change in \( v \) was negligible

• We should test that this is a good approximation

• To do this, take our result and find the change in velocity while the plane is skidding

• The total frictional force acting is \( 16\mu W_w \), so the plane’s acceleration is

\[ a = \frac{16\mu W_w}{M} \]
Therefore the change in velocity is:

$$\Delta v = \alpha t = \frac{16 \cdot 0.5 \cdot 142 \text{kN}}{395,000 \text{kg}} \cdot 0.25 \text{s} = 2.9 \text{m/s}$$

Not big, but maybe not negligible either!

To take this into account, we would modify our equation:

$$\alpha t = \frac{v}{R}$$

to read:

$$\alpha t = \frac{v(t)}{R} = \frac{v_0 - \alpha t}{R}$$