Relative Motion

- So far, we’ve studied kinematics in one, two, and three dimensions, with some special cases such as circular motion.
- Now we need to consider the nature of motion itself a little more closely.
- In everyday life, we have a sense of some things moving and others at rest.
  - E.g., a car moves down the freeway, but a billboard by the side of the road is at rest.
  - But, the car’s driver observes the car as being at rest, while the billboard moves past.
- Can we decide which description is “more true”?
  - No! We are used to treating the earth as a special, static “reference frame”, but in the grand scheme of the universe, it isn’t.
- Motion is a *relative* concept
  - The rules of kinematics should reflect this
- So what happens when we change reference frames:
  - If origin is $O$, displacement vector for object at $P$ is $\mathbf{r}_{PO}$
  - If origin is $O'$, displacement vector for object at $P$ is $\mathbf{r}_{PO'}$
  - We can translate from one coordinate system to the other using vector addition

\[
\mathbf{r}_{PO} = \mathbf{r}_{PO'} + \mathbf{r}_{O'O}
\]
• What if the two coordinate systems are moving with respect to one another?

• We can translate velocities, starting from the equation for translating positions:

\[ \mathbf{r}_{PO} = \mathbf{r}_{PO'} + \mathbf{r}_{O'O} \]

• Taking the derivative with respect to time gives:

\[ \frac{d\mathbf{r}_{PO}}{dt} = \frac{d\mathbf{r}_{PO'}}{dt} + \frac{d\mathbf{r}_{O'O}}{dt} \]

or,

\[ \mathbf{v}_{PO} = \mathbf{v}_{PO'} + \mathbf{v}_{O'O} \]
• Relative acceleration:
• Completely analogous to position and velocity:

\[ \mathbf{a}_{PO} = \mathbf{a}_{PO'} + \mathbf{a}_{O'O} \]

• But we’ll almost always deal with cases where the two frames have no relative acceleration (you’ll see why next lecture)
• In those cases,

\[ \mathbf{a}_{PO} = \mathbf{a}_{PO'} \]

– Acceleration is the same for all frames of reference moving at constant velocity with respect to each other
Example

What is the velocity of the car, if measured from the bicycle?

\[ \mathbf{v}_{\text{car}} = 20 \text{ m/s } \mathbf{i} \]

\[ \mathbf{v}_{\text{bicycle}} = 5 \text{ m/s } \mathbf{j} \]

- Both velocities listed in the diagram are measured with respect to the Earth
- We treat point \( O' \) as fixed to the Earth, and \( O \) as moving with the bicycle
- The Earth moves with respect to the bicycle at
  \[ \mathbf{v}_{O'O} = -5 \text{ m/s } \mathbf{j} \]
• We then apply our formula:

\[ \mathbf{v}_{PO} = \mathbf{v}_{PO'} + \mathbf{v}_{O'O} \]

where in this case \( \mathbf{v}_{PO} \) is the velocity of the car as seen by the bicycle and \( \mathbf{v}_{PO'} \) is the velocity seen from the Earth. So,

\[ \mathbf{v}_{PO} = \mathbf{v}_{\text{car,bicycle}} = 20\text{m/s} \mathbf{i} - 5\text{m/s} \mathbf{j} \]