Chapter 5:

Multiple choice:

8) 

(a) The maximum force exerted by static friction is $\mu_s N$. Since the block is resting on a level surface, $N = mg$. So the maximum frictional force is 

$$f = \mu_s mg = (0.80)(2.0\text{kg})(9.8\text{m/s}^2) = 15.7\text{N}.$$ 

Since this is more than the applied horizontal force, friction will exert as much force as it needs to neutralize the external force, and the block will not move. (B)

(b) With the block in motion, the force exerted by kinetic friction will be:

$$f = \mu_k N = \mu_k mg = (0.60)(2.0\text{kg})(9.8\text{m/s}^2) = 11.8\text{N}$$

This will be in the opposite direction opposite the motion, but is less than the applied external force. Hence the net force will be 1.9N in the direction of motion. (A)

12)

a) Since the speed of the motorcycle is constant as it goes around the loop, the magnitude of its acceleration, $\frac{v^2}{r}$, is also constant. Therefore the net force acting on the motorcycle also has constant magnitude. Since the net force is $\mathbf{N} + \mathbf{W} + \mathbf{f}$, the correct answer is (D).

b) Consider the forces acting on the motorcycle at a random point around the loop:

With these coordinates, the center of the loop is in the $y$ direction. The velocity is in the $x$ direction, and since it is constant we know there can be no acceleration in $x$, and therefore
no net force in that direction. \( N \) has no \( x \) component, and therefore the sum \( f + W \) must also have no \( x \) component. In other words, \( f + W \) is always directed toward the center of the circle. (C)

Questions:

7) By having the air exert a downward force, the normal force between the tires and the road is increased. This turn means that friction can exert a greater force on the car, allowing it to turn at a tighter radius than it could otherwise.

15) According to the equations for the velocity and period of the conical pendulum (5-14 and 5-15), the speed would be infinite and the period 0 for a 90° angle. This can never happen because if the string’s tension acts only in the horizontal direction, there will be nothing to balance the vertical force of gravity pulling the ball down.

For \( \theta = 0^\circ \), the velocity is zero. This is the case of the ball hanging straight down and rotating about its axis. There is no definite period associated with this type of motion.

16) As the turntable spins faster, the force between the coin and the turntable needed to keep the coin moving in a circle increases. This force is supplied by friction, so once it exceeds the maximum possible frictional force, the coin will move toward the outer edge of the turntable and eventually fall off.

Exercises:

13) We start as always by drawing a force diagram for the bar:

![Force diagram for the bar](image)

and for the mass:

![Force diagram for the mass](image)

Since we want the mass to be static, we must have:

\[
T_{1x} + T_{2x} = 0 \\
T_{1y} + T_{2y} = mg
\]
For the bar, the forces $N_1$ and $N_2$ are each equal to $F$ (from Newton’s Third Law), so they cancel. That leaves the net force equations as:

$$F_x = N_1 - N_2 = 0$$
$$F_y = f_1 + f_2 - mg$$

By symmetry, we know that $f_1 = f_2$. Each of these frictional forces has a value $\leq \mu_s F$, and since we’re interested in applying the least possible $F$, we’ll take the case $f_1 = f_2 = \mu_s F$. We want the net force in $y$ to be zero, so:

$$F_y = f_1 + f_2 - mg = 0$$
$$2\mu_s F = mg$$
$$F = \frac{mg}{2\mu_s} = \frac{75\text{kg} \cdot 9.8\text{m/s}^2}{2 \cdot 0.41} = 900\text{N}$$

26) We pull the crate at some angle, as shown below:

To find the normal force, we set the $y$ component of the net force to 0:

$$F_y = N + T \sin \theta - W = 0$$
$$N = W - T \sin \theta$$

So the largest force friction can apply is $f = \mu_s N = \mu_s (W - T \sin \theta)$. For the box to just start moving, the horizontal component of the tension must be equal to this force:

$$F_x = T \cos \theta - f = T \cos \theta - \mu_s (W - T \sin \theta) = 0$$
$$W = \frac{T \cos \theta}{\mu_s} + T \sin \theta$$

Now we need to know the angle to pull to maximize the weight we can move. This means taking the derivative with respect to $\theta$, and setting it equal to 0:
\[
\frac{dW}{d\theta} = T \left( -\sin \theta + \frac{\cos \theta}{\mu_s} \right) = 0
\]
\[
\sin \theta = \mu_s \cos \theta
\]
\[
\tan \theta = \mu_s
\]
\[
\theta = \tan^{-1} \mu_s = \tan^{-1} 0.35 = 19^\circ
\]

Putting this into our equation for \( W \), we find that the maximum weight that can be moved is:
\[
W = \frac{T \cos \theta}{\mu_s} + T \sin \theta = 1.22\text{kN} \left( \frac{\cos 19^\circ}{0.35} + \sin 19^\circ \right) = 3.69\text{kN}
\]

30) First we need to determine the frictional force acting on the block. In this case friction acts on two of the block’s surfaces, and we need to find the normal force for each. To do so, let’s look at the side view first:

Since the block is not accelerating in the \( z \) direction, \( N \) (the net normal force) must be equal to \( mg \cos \theta \).

Now consider the front view:

By symmetry, we know that \( |N_1| = |N_2| \), and that the \( y \) components are equal and opposite. So the net normal force is:
\[ N = |N_1|\cos 45^\circ + |N_2|\cos 45^\circ = mg \cos \theta \]
\[ \frac{2}{\sqrt{2}} |N_1| = mg \cos \theta \]
\[ |N_1| = \frac{mg \cos \theta}{\sqrt{2}} = |N_1| \]

So the friction force acting on each surface is \( f_{1,2} = \mu_k N_{1,2} = \frac{\mu_k mg \cos \theta}{\sqrt{2}} \), which means the total frictional force is \( f = f_1 + f_2 = \sqrt{2}\mu_k mg \cos \theta \). From the side-view diagram, this makes the net force in \( y \) equal to
\[ mg \sin \theta - \sqrt{2}\mu_k mg \cos \theta = mg \left( \sin \theta - \sqrt{2}\mu_k \cos \theta \right). \]

Therefore the acceleration is \( g \left( \sin \theta - \sqrt{2}\mu_k \cos \theta \right). \)

33) The forces on the car are (looking from the rear of the car):

The frictional force \( f \) is the only force acting in \( x \), so it alone must provide enough force to accelerate the car in circular motion.
a) The force needed is:

\[ F = \frac{mv^2}{r} = \frac{(10.7\text{ kN}/9.8\text{ m/s}^2) \cdot (13.4\text{ m/s})^2}{61.0\text{ m}} = 3.2\text{ kN} \]

b) Since the normal force is equal to the car’s weight of 10.7kN, the minimum coefficient of static friction needed is:

\[ f = \mu_s N \]
\[ \mu_s = \frac{f}{N} = \frac{3.2\text{ kN}}{10.7\text{ kN}} = 0.30 \]

Problems:

3) From Newton’s Second Law, we know that the acceleration of the system will be

\[ a = \frac{F}{2m} \hat{j} \]

where I’ve taken the y direction to be the direction of the force. By symmetry, we also know that each particle will have the same y acceleration and equal but opposite x accelerations.

Now consider the forces acting on one particle:

\[ a_y = \frac{T \sin \theta}{m} = \frac{F}{2m} \]
\[ T = \frac{F}{2 \sin \theta} \]
\[ a_x = \frac{T \cos \theta}{m} = \frac{F \cos \theta}{2m \sin \theta} = \frac{F}{2m} \tan \theta \]

The fact that the particle is held by the string means that:

\[ x^2 + y^2 = L \]
\[ \tan \theta = \frac{y}{x} = \frac{\sqrt{L^2 - x^2}}{x} \]
Substituting this into the equation for \( a_x \) gives:

\[
    a_x = \frac{F}{2m} \tan \theta = \frac{F}{2m} \left( \frac{x}{\sqrt{L^2 - x^2}} \right)
\]

The case \( x = L \) is the situation shown in Fig. 5-46. In this case, any nonzero \( F \) will result in an infinite acceleration in \( x \), meaning that the tension in the string will be infinite (for that instant). Any real string would stretch under these circumstances, making \( \sqrt{x^2 + y^2} > L \), since applying an infinite tension is impossible.

11) The key to this problem is to realize that the tension in the string is different on the two sides of the dowel. To see why, consider the forces acting on a small segment of the string on the dowel (this segment subtends an angle of \( d\theta \)):

Each tension vector is at an angle of \( \frac{d\theta}{2} \) with respect to the \( x \) axis.

Assume the system is at rest. Then the forces must balance:

\[
    F_y = N - T_1 \sin \frac{d\theta}{2} - T_2 \sin \frac{d\theta}{2} = 0
\]

\[
    F_x = T_1 \cos \frac{d\theta}{2} + f - T_2 \cos \frac{d\theta}{2} = 0
\]

Since we know that \( d\theta \) is a very small angle, we have the approximate relations:

\[
    N = T_1 \frac{d\theta}{2} + T_2 \frac{d\theta}{2}
\]

\[
    T_1 + f = T_2
\]

\[
    T_1 + \mu (T_1 + T_2) \frac{d\theta}{2} = T_2
\]

\[
    T_2 - T_1 = dT = \mu (T_1 + T_2) \frac{d\theta}{2}
\]
Since $dT$ is small, we can approximate $T_1 + T_2$ as $2T$, where $T$ is the average of the two tensions. So we have:

$$d\theta = \frac{dT}{\mu T}$$

To find the total change in tension between the two sides of the string, we integrate over the entire length of contact (from $\theta = 0$ to $\pi$):

$$\int_0^\pi d\theta = \pi = \int_0^{\tau_f} \frac{dT}{\mu T} = \frac{1}{\mu} \ln T|_{\tau_i}^{\tau_f} = \frac{1}{\mu} \ln \frac{T_f}{T_i}$$

In our case, we want the initial tension to balance $W$ and the final tension to balance $F$, so:

$$\pi = \frac{1}{\mu} \ln \frac{F}{W}$$

$$\frac{F}{W} = e^{\mu \pi}$$

$$F = We^{\mu \pi}$$

17)

a) The forces on the ball are:

There is no acceleration in $y$, so:

$$T_1 \cos \theta_1 = T_2 \cos \theta_2 + mg$$

$$T_2 = \frac{T_1 \cos \theta_1 - mg}{\cos \theta_2} = \frac{35 \text{N} \cos 60^\circ - 1.34 \text{kg} \cdot 9.8 \text{m/s}^2}{\cos 60^\circ} = 8.74 \text{N}$$

b) The net force is in the $x$ direction:
\[ F_x = -T_1 \sin \theta_1 - T_2 \sin \theta_2 = -35 \text{N} \sin 60^\circ - 8.74 \text{N} \sin 60^\circ = -37.9 \text{N} \]

c) The net force accelerates the ball in a circle, so its magnitude must be equal to \( \frac{mv^2}{r} \).

The radius of the circle is given by:

\[ \tan 60^\circ = \frac{r}{\frac{d}{2}}, \text{ where } d \text{ is the distance between where the two ropes are attached to the rod} \]
\[ r = \frac{1.70 \text{m} \tan 60^\circ}{2} = 1.47 \text{m} \]

Therefore, the velocity is given by:

\[ \frac{mv^2}{r} = 37.9 \text{N} \]
\[ v = \sqrt{\frac{37.9 \text{N} \cdot 1.47 \text{m}}{1.34 \text{kg}}} = 6.44 \text{m/s} \]

Chapter 6:

Multiple choice:

3) The form of Newton’s Second Law as written by Newton tells us that the rate of change of the momentum of an object is equal to the force acting on it. For circular motion, the force (and therefore the rate of change of momentum) is proportional to \( v^2 \). (C)

7) If the impulse is 0, then the change in momentum must also be 0. However, it is possible for the position of the object to change. For example, consider an object at rest, acted on by a force of 1N in the +x direction for 1s, and then by a force of 1N in the –x direction. The first force will cause the object to accelerate in the +x direction, and the second will bring it to rest again, but won’t move it back to \( x = 0 \). So \( \Delta p \) is always 0, but \( \Delta r \) may not be. (C)

10) As far as we know, the law of conservation of momentum is always valid. (A)

Questions:

3) The initial velocity imparted to the baseball depends on the batter. This means that some batters are able to cause a greater change in the momentum of the pitched ball than others can, by applying a greater impulse during the ball-bat collision.
8) There are two competing effects at work here. For one thing, the scale is not supporting the grains of sand that are falling, so that tends to reduce the reading. On the other hand, the sand striking the pile on the bottom exerts an impulse on the bottom pile, and that impulse must be balanced by the scale. This effect tends to increase the scale’s reading.

We know that each grain of sand begins at rest in the top half of the glass, and ends at rest in the bottom half, so the net impulse acting on it must have been 0. This means that the two effects discussed above exactly cancel, so the hourglass weighs the same whether the sand is falling or not.

Exercises:

2) If we take North to be the +y direction and East to be the +x direction, we have:

Initial momentum = \( m \dot{v} = 2000 \text{kg} \left( 40 \frac{\text{km}}{\text{hr}} \right) = 8 \times 10^4 \text{kg} \frac{\text{km}}{\text{hr}} \)

Final momentum = \( 2000 \text{kg} \left( 50 \frac{\text{km}}{\text{hr}} \right) = 1.0 \times 10^5 \text{kg} \frac{\text{km}}{\text{hr}} \)

So the change in momentum is:

\[ p_f - p_i = 1.0 \times 10^5 \text{kg} \frac{\text{km}}{\text{hr}} \hat{i} - 8 \times 10^4 \text{kg} \frac{\text{km}}{\text{hr}} \hat{j} \]

12)

a) Since the hand travels 2.8cm before coming to rest, and we are assuming constant force on it, we can determine the time from:

\[ \Delta x = v_o t - \frac{1}{2} a t^2 \]
\[ \Delta v = 0 - v_o = -at \]
\[ \Delta x = v_o t - \frac{1}{2} \left( \frac{v_o}{t} \right)^2 = \frac{1}{2} v_o t \]
\[ t = \frac{2\Delta x}{v_o} = \frac{2 \times 0.028 \text{m}}{9.5 \text{m/s}} = 5.9 \times 10^{-3} \text{s} \]

b) We know that the impulse equals the change in momentum of the hand. So,
\[
F_{\text{avg}} \Delta t = \Delta p = m \Delta v = m(v_o - 0)
\]
\[
F_{\text{avg}} = \frac{mv_0}{\Delta t} = \frac{0.540 \text{kg} \cdot 9.5 \text{m/s}}{5.9 \times 10^{-3} \text{s}} = 870 \text{N}
\]

18) The initial momentum of the car + man system as measured by an observer next to the tracks is

\[
p_i = (M_{\text{car}} + m_{\text{car}})v_o = \frac{v_0}{g}(W + w)
\]

After the man starts running the same observer measures the car’s velocity as \( v' \) and the man’s velocity as \( v' - v_{\text{rel}} \). Therefore, the final momentum is measured as:

\[
p_f = \frac{Wv'}{g} + \frac{w(v' - v_{\text{rel}})}{g}
\]

Since no external forces act on the system, \( p_i = p_f \), or:

\[
\frac{v_0}{g}(W + w) = \frac{Wv'}{g} + \frac{w(v' - v_{\text{rel}})}{g}
\]
\[
v'(W + w) = v_0(W + w) + wv_{\text{rel}}
\]
\[
v' = \frac{v_0(W + w) + wv_{\text{rel}}}{W + w}
\]
\[
\Delta v = v' - v_0 = \frac{wv_{\text{rel}}}{W + w}
\]