Chapter 7:

Questions:

17) Yes they do. To see why, consider the case (Case 1) where they all jump at once. Just before they leap off the car, they’ve already given it its final velocity. The final state then consists of the people moving at $v_{\text{rel}} + v_{\text{car}}$ and the car moving at $v_{\text{car}}$ (in the opposite direction).

Now consider the case (Case 2) where one person jumps off early. His mass is much smaller than the entire group’s (and much smaller than the mass of the car), so he imparts a very small velocity to the car by jumping. His final speed with respect to the ground is greater than the entire group’s final speed in the above case, and is in fact very close to $v_{\text{rel}}$. Now suppose everybody else jumps off, and assume the car has the same final velocity as in Case 1. Thus the final system consists of the car moving at $v_{\text{car}}$, N-1 people moving at $v_{\text{rel}} + v_{\text{car}}$, and the one who jumped earliest moving at some greater speed. But this violates conservation of momentum! (Remember that momentum was conserved in Case 1 when all the people had final velocity $v_{\text{rel}} + v_{\text{car}}$.) Thus the car must be moving just a little faster in case 2 than in case 1. For the case where the people jump off one at a time, the same argument holds for each one, so the final velocity of the car can be much greater than it was in case 1.

The lesson here is that if you’re designing a rocket and want to achieve the largest possible final speed, it’s better to expel your fuel a little at a time rather than all at once!

22) Yes it can. One simple example is to consider a rocket with more than half its initial mass comprised of fuel. If the rocket is initially at rest, and then expels all the fuel instantly at $v_{\text{rel}}$, the rocket’s final velocity must be greater than $v_{\text{rel}}$ to conserve momentum.

Exercises:

20)

a) From Equation 7-31, we find that the magnitude of the thrust is given by

$$v_{\text{rel}} \frac{dm}{dt} = 3.27 \text{km/s} \cdot 480 \text{kg/s} = 157000 \text{km} \cdot \text{kg/s}^2 = 1.57 \times 10^8 \text{N}$$
b) The total amount of fuel burned is $480\text{kg/s} \cdot 250\text{s} = 1.20 \times 10^5\text{kg}$. Thus the remaining mass of the rocket is $2.55 \times 10^5\text{kg} - 1.20 \times 10^5\text{kg} = 1.35 \times 10^5\text{kg}$.

c) Equation 7-31 tells us that $M \frac{dv}{dt} = v_{\text{rel}} \frac{dM}{dt}$ (I use a scalar rather than vector equation since the motion is one-dimensional in this problem). Thus we have:

$$M \frac{dv}{dt} = v_{\text{rel}} \frac{dM}{dt}$$

$$dv = v_{\text{rel}} \frac{dM}{M}$$

$$\int_{v_i}^{v_f} dv = \int_{M_i}^{M_f} v_{\text{rel}} \frac{dM}{M}$$

$$v_f - v_i = v_{\text{rel}} \ln \frac{M_f}{M_i} = -3.27\text{km/s} \cdot \ln \frac{1.35 \times 10^5\text{kg}}{2.55 \times 10^5\text{kg}} = 2.08\text{km/s}$$

Problems:

11) The equation for variable-mass systems (7-31) is applicable to both barges in this problem. Also, the problem is only concerned with the one-dimensional motion of each barge – the assumption is that the rudder of each is adjusted such that to force from the water keeps them moving in a straight line. With this simplification, equation 7-31 becomes:

$$M \frac{dv_x}{dt} = F_{\text{net,x}} + v_{\text{rel,x}} \frac{dM}{dt}$$

We want both barges to move at constant velocity, so $\frac{dv_x}{dt} = 0$, and therefore:

$$F_{\text{net,x}} + v_{\text{rel,x}} \frac{dM}{dt} = 0$$

Let’s analyze the slower barge first. It is losing mass as the coal is shoveled out, but since the coal is being shoveled exactly sideways $v_{\text{rel,x}} = 0$, and thus $F_{\text{net,x}} = 0$. In other words, the engine does not need to apply a force.

The faster barge is gaining mass, and the incoming coal is moving slower than the barge, with a relative velocity of $9.65\text{km/h} - 21.2\text{km/h} = -11.55\text{km/h}$. So we do need the engines to supply a force of:
\[ F_{\text{net},x} = -v_{\text{rel},x} \frac{dM}{dt} = 11.55\text{km/h} \cdot 925\text{kg/min} \]
\[ = 3.21\text{m/s} \cdot 15.4\text{kg/s} = 49.5\text{N} \]

Chapter 11:

Multiple choice:

1) Throughout the book’s motion, the student applies a force that prevents it from freely falling to the floor. This force must be upwards, which is opposite to the direction of motion. Therefore, the work is negative. (C)

6) The maximum speed of the car is the point at which drag forces exactly cancel the maximum force the engine can supply. Since the drag forces are proportional to speed, we have:

\[ F_{\text{engine}} = av_{\text{max}} \]

where \( a \) is some constant. The power delivered by the engine at this velocity is given by:

\[ P = F_{\text{engine}}v_{\text{max}} = av_{\text{max}}^2 \]

\[ v_{\text{max}} = \sqrt{\frac{P}{a}} \]

From this equation, we see that if \( P \) is doubled, \( v_{\text{max}} \) will increase by a factor of \( \sqrt{2} \). (B)

8)

a) We find the point where the maximum force is required by differentiating, and setting the result to 0:

\[ \frac{dF}{dx} = 2kx - kl = 0 \]

\[ x = \frac{l}{2} \]

(D)

b) In principal this is a two-step problem. First we need to find how the work varies with \( x \), and then differentiate to find the maximum. However, we know that:

\[ W = \int F \cdot dx \]
and since all the motion and forces are in one dimension, this simplifies to:

\[ W = \int F \, dx \]

Therefore, \( \frac{dW}{dx} = -F(x) = -kx(x - l) \). [The minus sign comes in because the work we are interested in is work done by a force acting on the spring, while \( F(x) \) is the force supplied by the spring. In other words, to compress the spring we must apply a force of (at least) \(-F(x)\).] There are two values of \( x \) for which this is 0 (\( x = 0 \) and \( x = l \)). To see which is the maximum, we need to evaluate the second derivative of \( W \) at each of these values:

\[
\frac{d^2W}{dx^2} = -2kx + kl \\
= kl \text{ when } x = 0 \\
= -kl \text{ when } x = l
\]

Thus we see that the maximum work occurs when \( x = l \). (E).

12) Power is the rate at which work is done, or equivalently the force applied to an object multiplied by the object’s velocity. In this problem, both cars experience the same force, of magnitude \( \frac{a}{m} \). However, since the lighter car has the greater acceleration, it will also have the greater velocity at any time after starting; hence its engine delivers the most power (A).

Questions:

3) We begin with the general definition of work:

\[ W = \int_{r_1}^{r_2} F \cdot dr \]

If \( F \) is the sum of three forces, we can write:

\[
W = \int_{r_1}^{r_2} \left( F_1 + F_2 + F_3 \right) \cdot dr = \int_{r_1}^{r_2} \left( F_1 \cdot dr + F_2 \cdot dr + F_3 \cdot dr \right) \\
= \int_{r_1}^{r_2} F_1 \cdot dr + \int_{r_1}^{r_2} F_2 \cdot dr + \int_{r_1}^{r_2} F_3 \cdot dr \\
= W_1 + W_2 + W_3
\]
This proves that the total work is the sum of the work done by each force.

8) There’s nothing wrong. The force you apply to lift the ball also does work on it, as does the force of gravity pulling the ball down. In this case the work you do is positive, and the work gravity does is negative, and the total work is 0 (as it must be, since the total force acting on the ball is 0).

11) To lift the books you must supply a force at least as large as gravity does. Since the force of gravity is the weight of the books, and the work you must do is proportional to the force you must apply, the work does depend on the weight of the books. The work does not depend on the mass of the books (in deep space, where there is no gravity, it takes no work to move the books).

Since work is also proportional to the distance over which a force acts, it will depend on the height of the upper shelf. However, it won’t depend on the time or path the books take going from one shelf to the other, since \( W = F \cdot \Delta r \) only depends on the force, and the initial and final positions.

19) We take object 1 to be less massive than object 2. Equal kinetic energies means that:

\[
\begin{align*}
m_1v_1^2 &= m_2v_2^2 \\
v_1 &= v_2\sqrt{\frac{m_2}{m_1}}
\end{align*}
\]

Object 1’s momentum is \( m_1v_1 = m_1\sqrt{\frac{m_2}{m_1}}v_2 = \sqrt{m_1m_2v_2} \). Since \( m_1 \) is less than \( m_2 \), this is less than \( m_2v \). Therefore the less massive object has less momentum than the more massive one.
Exercises:

1) The force diagram is:

The displacement of the box while these forces act is 3.3mi.

a) The work done by the worker is:

\[ W = F \cdot \Delta r = |F| |\Delta r| \cos \theta = 190N \cdot 3.3m \cdot \cos 22^\circ \]
\[ = 581J \]

b) Gravity acts only in the vertical direction, which in this case is perpendicular to the motion. Therefore, gravity does no work.

c) Similarly, the normal force does no work, since it is also perpendicular to the motion.

17) 

a) One kilowatt-hour is 3.6 x 10^6J, or 3.6MJ. Since each gallon of gas provides 140MJ of energy, this corresponds to 0.025 gallons. Therefore the distance the car can travel while using this much energy is 2.5% of the distance it could travel on a gallon of gas, or:

\[ d = 0.025 \cdot 30mi = 0.77mi \]

b) If the car travels at 55mi/hr, it is burning 1.83 gallons of gas per hour. Thus the rate of energy expenditure is:

\[ P = \frac{E}{t} = \frac{1.83\text{gallons} \cdot 140\text{MJ/gallon}}{3600\text{s}} = 71.2\text{kW} \]

21) The graph shows that the acceleration is increasing linearly with position:

\[ a = sx + b \]
The intercept $b$ is zero, and the slope $m$ is given by:

\[ s = \frac{\Delta a}{\Delta x} = \frac{20\text{m/s}^2}{8\text{m}} = 2.5\text{s}^2 \]

Thus,

\[ a(x) = \frac{2.5}{s^2} x \]

\[ F(x) = ma(x) = 10\text{kg} \cdot \frac{2.5}{s^2} x \]

The work can now be found by integration:

\[
W = \int_{x_i}^{x_f} F(x) \, dx = \int_{0\text{m}}^{8\text{m}} 10\text{kg} \cdot \frac{2.5}{s^2} x \, dx
\]

\[
= \frac{25\text{kg}}{s^2} \left( \frac{x^2}{2} \right)_{0\text{m}}^{8\text{m}} = 800\text{J}
\]

Problems:

3)

a) Since there is no friction, the tension in the cable is the same, and equal to $F$, everywhere. If we assume that the load is being lifted as a constant velocity, then nothing in the system is accelerating. We can now draw the force diagram for the bottom two pulleys:

Thus the net force acting on the bottom two pulleys is:
\[ F_{\text{net}} = 4T - g \left( m_p + m_L \right) = 4F - g \left( m_p + m_L \right) \]

The load can be lifted by applying a force large enough to make the net force 0:

\[
F_{\text{net}} = 4T - g \left( m_p + m_L \right) = 4F - g \left( m_p + m_L \right) \\
F = \frac{g \left( m_p + m_L \right)}{4} = \frac{860.0 \text{lb}}{4} = 215 \text{lb}
\]

b) The work done against gravity required to lift the load alone 12ft is:

\[
W = mgh = 840 \text{lb} \cdot 12 \text{ft} = 1.01 \times 10^4 \text{ft-lb}
\]

c) We need to determine, for a given length \( L \) of cable pulled down by \( F \), how much the load rises. Do do this, consider the total length \( S \) of the cable in terms of the portion to the right of the upper pulley \( (y_F) \) and the \( y \) positions of the four pulleys (I number the pulleys from top to bottom):

\[
S = y_F + y_4 - y_i + y_4 - y_2 + y_3 - y_2 + y_3 - y_2 \\
= y_F + 2y_4 + 2y_3 - y_1 - 3y_2
\]

After the length \( L \) is pulled, we have:

\[
S' = y_F + L + 2y'_4 + 2y'_3 - y_1 - 3y_2
\]

(since only pulleys 3 and 4 are free to move). But the total length of the cable doesn’t change, so:

\[
y_F + L + 2y'_4 + 2y'_3 - y_1 - 3y_2 = y_F + 2y_4 + 2y_3 - y_1 - 3y_2 \\
L = 2(y_4 - y'_4) + 2(y_3 - y'_3)
\]

Since pulleys 3 and 4 are attached, each must have the same change in position, and this is equal to the distance the load is raised:

\[
L = 4\Delta y
\]

Since we want \( \Delta y \) to be 12ft in this case, the force must be applied for 48ft.

d) The work done by the force is \( F\Delta y = 215 \text{lb} \cdot 48 \text{ft} = 1.03 \times 10^4 \text{ft-lb} \).

10)

a) We differentiate to find the speed that gives the maximum power:
\[ \frac{dP}{dv} = ab - 3av^2 = 0 \]
\[ b = 3v^2 \]
\[ v = \sqrt{\frac{b}{3}} \]

b) The force applied is given by \( F = \frac{P}{v} = a(b - v)^2 \). Again we differentiate to find the maximum:
\[ \frac{dF}{dv} = -2av = 0 \]
\[ v = 0 \]

c) Even though the power is zero when the velocity is 0, the force applied is not (in fact, this is the point where the motor can apply the largest force). Therefore the trolley is able to start moving from rest.

13) The work required to stretch the spring an arbitrary distance \( s \) is:

\[ W = \int_0^s F(x) \, dx = \int_0^s kx^3 \, dx = \frac{kx^4}{4} \]

We know that the work done in stretching the spring to \( l \) is \( W_o \):

\[ W_o = \frac{kl^4}{4} \]

So to stretch the spring to \( 2l \) the work done must be:

\[ W' = \frac{k(2l)^4}{4} = 16 \cdot \frac{kl^4}{4} = 16W_o \]

The additional work required to go from \( l \) to \( 2l \) is therefore \( 15W_o \).