Lecture 14: Collisions

- Last lecture, we learned about impulse, and gave a couple of examples of how it could be applied to a pair of objects that interact strongly for a short time.
- Such a powerful, brief interaction is known as a *collision*, and analyzing collisions is one of the main applications of conservation of momentum.
• Consider a generic collision between two objects:

Before:

\[ m_1 \quad v_{1i} \]

\[ m_2 \quad v_{2i} \]

After:

\[ v_{1f} \]

\[ v_{2f} \]

• If we know the masses and initial velocities, can we predict the final velocities?
• The initial momentum is given by:

\[ p_i = m_1 v_{1i} + m_2 v_{2i} \]
\[ p_f = m_1 v_{1f} + m_2 v_{2f} \]

• Since the only forces acting are between the two objects (and therefore internal to the system), the total momentum is conserved, so:

\[ p_i = p_f \]
\[ m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \]

• We have two unknown velocities and only one equation
  – The answer will depend on the details of the interaction between the two objects
  – But if we know one of the final velocities, we can find the other
Example: Frictionless Pool Table

• Imagine playing pool on a frictionless table. You shoot the cue ball toward the 8 ball:

  - After the collision, the cue ball has the following velocity:

• Does the 8 ball go in the pocket?
• From conservation of momentum we have:

\[ p_{x,i} = m v_{x1,i} = p_{x,f} = m v_{x1,f} + m v_{x2,f} \]

\[ p_{y,i} = 0 = p_{x,f} = m v_{y1,f} + m v_{y2,f} \]

\[ v_{x2,f} = \frac{m v_{x1,i} - m v_{x1,f}}{m} = v_{x1,i} - v_{x1,f} = -2 \text{m/s} + 0.7 \text{m/s} \cdot \cos 45^0 \]

\[ = -1.51 \text{m/s} \]

\[ v_{y2,f} = \frac{-m v_{y1,f}}{m} = 0.49 \text{m/s} \]

• So the 8 ball moves at an angle of:

\[ \theta = \tan^{-1} \frac{v_{y2,f}}{v_{x2,f}} = 18^0 \]

• Therefore it hits the pocket
Completely Inelastic Collisions

• In some collisions, the two colliding objects become “stuck” and move off together with the same velocity
• For such *completely inelastic* collisions we can determine the final velocity from conservation of momentum:

\[
p_i = p_f
\]

\[
m_1 v_{1i} + m_2 v_{2i} = m_1 v_f + m_2 v_f
\]

\[
v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}
\]
Example: Loading a Coal Train

- Hopper cars roll under a chute and coal is deposited:
The hopper car has an empty weight of 25 tons, and is loaded with 100 tons of coal. If it rolls without friction at 1m/s into the loading area, what is its speed after being loaded?

This is an example of a completely inelastic collision
  - There is an interaction between the car and the coal, after which they move off together

We use conservation of momentum:

\[
\mathbf{p}_i = m_{\text{car}} \mathbf{v}_i = m_{\text{car}} v_i \mathbf{i}
\]

\[
\mathbf{p}_f = m_{\text{car+coal}} \mathbf{v}_f = m_{\text{car+coal}} v_f \mathbf{i}
\]

\[
v_f = \frac{v_i m_{\text{car}}}{m_{\text{car+coal}}} = \frac{1 \text{m/s} \cdot (25 \text{tons/g})}{125 \text{tons/g}} = 0.20 \text{m/s}
\]
Further Analysis

• What about momentum conservation in the y direction?
  – Just before striking the car, the coal’s momentum is downward
  – In the final situation, the momentum of the coal + car system is only in the x direction
  – Does this violate conservation of momentum?

• No!
  – There is an external force on the coal+car system – the normal force from the rails pushing up on the car
  – Since this external force is only in the y direction, our use of conservation of the x component of momentum is valid
Consistency with Newton’s Laws

• Our analysis indicates that the hopper car slows down as it is loaded with coal.
• By Newton’s Second Law, this means some force must be acting on the car. What force is this?
  – The coal originally has no horizontal velocity
  – By interacting with the car, it gains a final velocity of 0.20m/s. So, we know the car applies a force to the coal
  – By Newton’s Third Law, the coal applies a force of equal magnitude but opposite direction to the car
  – It’s that force that slows down the hopper car.
Center-of-Mass Reference Frame

• It is often simplest to analyze collisions in a frame in which the total initial momentum is 0. Can such a frame always be found?

• General case:

\[ \mathbf{p}_i = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 \]

• We want to find a new frame in which the momentum is 0. The velocity of the new frame with respect to the original one is:

\[ m_1 (\mathbf{v}_1 - \mathbf{v}_{CM}) + m_2 (\mathbf{v}_2 - \mathbf{v}_{CM}) = 0 \]

\[ (m_1 + m_2) \mathbf{v}_{CM} = m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 \]

\[ \mathbf{v}_{CM} = \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2}{m_1 + m_2} \]
• Note that $v_{\text{CM}}$ is the same as the final velocity for completely inelastic collisions
  – Makes sense, since the final velocity in the center-of-mass frame must be zero (since momentum is conserved, and initial momentum was zero in that frame)