Lecture 8: Mass and Forces

• Recall that we originally defined mass as a measure of an object's resistance to acceleration
  – Mass defined this way is called “inertial mass”

• But now it seems that we might also define mass as a measure of the force of gravity on an object
  – Mass defined this way is called “gravitational mass”

• Question: are these two definitions guaranteed to give the same answer?
  – Experiments indicate that the answer is yes (within the precision of the measurements)
  – In Einstein’s theory of General Relativity, the two must be exactly equal
  – But more sensitive experiments to look for slight discrepancies are still going on
Mass: What We Know – and Don’t Know

• The origin of mass is (still) being explored by physicists
  – Any object we see in everyday life is made up of a whole lot of atoms
  – Atoms, in turn, are made up of electrons, protons, and neutrons
  – Protons and neutrons are made up of quarks
  – As far as we know, electrons and quarks are fundamental particles

• So, we might think of the mass of an object as just a count of how many fundamental particles are in it…
  – …but where do the fundamental particles get their mass?
• Until the mid ’60s, this was a total mystery
  – One couldn’t even formulate a mathematically sensible theory in which electrons or quarks had any mass at all!
• Finally resolved by the invention of the “Higgs mechanism”
• In this picture, electrons and quarks become massive by continuously interacting with a particle known as the Higgs boson
• A neat idea…
  – …but no one’s seen a Higgs boson yet!
• So, the physics community (literally thousands of us!) is actively looking for it
LHC – The Next Big Experiment in Particle Physics
The ATLAS Detector
• If the Higgs boson is indeed found, we’ll know why electrons and quarks have non-zero masses
• But, why do they have the particular masses that they do? 
  – e.g., why did the electron “choose” a mass of $9.1 \times 10^{-31}$ kg?
• Nobody knows!

There are plenty of mysteries left in the universe
Applications of Newton’s Laws

• Newton’s Laws tell us how an object will move if one knows the forces acting on it
• What types of forces are there?
• Fundamentally, we now believe that there are four types of forces in the universe:
  1. Gravity
     • You know how this one behaves (at least near the Earth’s surface)
  2. Electromagnetism
     • It’s what binds electrons to protons to make atoms; binds atoms to each other to make molecule, and so on
     • All the forces you “feel” in everyday life are due to either gravity or electromagnetism
  3. The strong nuclear force
  4. The weak nuclear force
     • Just as “important” as the first two, but not as readily apparent in everyday life
“Strength” of the Fundamental Forces

- If we consider two quarks near each other \((10^{-18}\text{m} \text{ apart, a typical distance within a proton or neutron})\), the fundamental forces act on it with the following strengths:

<table>
<thead>
<tr>
<th>Force</th>
<th>Relative Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>25</td>
</tr>
<tr>
<td>Electromagnetic</td>
<td>1</td>
</tr>
<tr>
<td>Weak</td>
<td>0.8</td>
</tr>
<tr>
<td>Gravity</td>
<td>(10^{-41})</td>
</tr>
</tbody>
</table>

- Gravity is extremely weak – but it is always attractive and acts over long distances
  - So it has a big effect on the universe!
Motion Under Gravity

- Near the Earth’s surface, any particle acted on by only gravity will have an acceleration of

\[ \mathbf{a} = -9.8 \text{m/s}^2 \mathbf{j} \]

or, \( a_x = 0, \ a_y = -9.8 \text{m/s}^2 \), where we’ve taken \( +y \) to be the upwards direction

- In other words, any object flying freely through the air will have a \textit{constant} velocity in the horizontal direction, and a velocity in the vertical direction that decreases at the rate of \( 9.8 \text{m/s}^2 \)
Projectile Trajectory

• Consider an object launched into the air with initial speed $v_o$, and initial direction $\phi_o$ from the horizontal:

• The initial $x$ velocity is $v_o \cos \phi_o$, and the initial $y$ velocity is $v_o \sin \phi_o$
• At later times, the velocity components are given by:

\[ v_x(t) = v_{o,x} + a_x t \]
\[ v_y(t) = v_{o,y} + a_y t \]

• But we know that \( a_x = 0 \) and \( a_y = -g \), so:

\[ v_x(t) = v_{o,x} + 0 = v_o \cos \phi_o \]
\[ v_y(t) = v_{o,y} - gt = v_o \sin \phi_o - gt \]

• This also allows us to find the position at any time:

\[ x(t) = x_o + v_o t \cos \phi_o \]
\[ y(t) = y_o + v_o t \sin \phi_o - \frac{1}{2} gt^2 \]
• For convenience, we can take $x_0 = y_0 = 0$. Rearranging the equations from the previous slide gives:

$$t = \frac{x}{v_o \cos \varphi_o}$$

$$y = v_0 t \sin \varphi_o - \frac{1}{2} g t^2$$

$$= x \frac{\sin \varphi_o}{\cos \varphi_o} - \frac{1}{2} g \frac{x^2}{v_o^2 \cos^2 \varphi_o}$$

• This is the equation of a *parabola*
• Using this, we can find the range of the object – how far it will go before returning to it’s original level of \( y = 0 \).
• Need to solve:

\[
x \frac{\sin \varphi_o}{\cos \varphi_o} - \frac{1}{2} g \frac{x^2}{v_o^2 \cos^2 \varphi_o} = 0
\]

• Note that \( x = 0 \) is a solution, but not what we want
  – This just means that the initial position was \( x = y = 0 \)
• The other solution is more interesting:

\[
x = \frac{2v_o^2 \cos^2 \varphi_o}{g} \cdot \frac{\sin \varphi_o}{\cos \varphi_o} = \frac{2v_o^2 \cos \varphi_o \sin \varphi_o}{g} = \frac{v_o^2 \sin 2\varphi_o}{g}
\]
• Is we are limited to a certain initial velocity, at what angle should we launch to achieve maximum range?
  – Range is maximum when $\sin 2\phi_o = 1$
  – Means $\phi_o = 45^\circ$

• What about the case when we’re shooting uphill?

• Now the object will land not when $y = 0$, but when it intersects the line $y = x \tan \theta$
• So we want

\[ y = x \frac{\sin \phi_o}{\cos \phi_o} - \frac{1}{2} g \frac{x^2}{v_o^2 \cos^2 \phi_o} = x \tan \theta \]

\[ \tan \phi_o - \tan \theta = \frac{1}{2} g \frac{x}{v_o^2 \cos^2 \phi_o} \]

\[ x = \frac{2v_o^2 \cos^2 \phi_o}{g} (\tan \phi_o - \tan \theta) \]

• Note that if \( \theta = \phi_o \), \( x = 0 \), just as it should
Example: Throwing out a baserunner

- A third baseman needs to throw the ball to first base, 120ft away. Sometimes reporters will suggest that the ball is thrown “on a line” (e.g., with no initial vertical velocity). If so, what velocity is needed if the ball is thrown from a height of 7ft and reaches first base without bouncing?
- First, we need to find how long it would take the ball to drop 7ft:

\[ y = y_o + v_{y,o} t - \frac{1}{2} gt^2 \]

\[ 0\text{ft} = 7\text{ft} + 0t - \frac{1}{2} \left(32\text{ft/s}^2\right) t^2 \]

\[ 16\text{ft/s}^2 t^2 = 7\text{ft} \]

\[ t = 0.66\text{s} \]
• Now we look at the motion in $x$:

$$x = v_{o,x} t$$

$$120\text{ft} = v_{o} (0.66\text{s})$$

$$v_{o} = 182 \frac{\text{ft}}{\text{s}} \cdot \frac{1\text{mi}}{5280\text{ft}} \cdot \frac{3600\text{s}}{1\text{hr}} = 124\text{mi/hr}$$

• That’s not humanly possible – the reporters are wrong!