Lecture 9: Projectile Examples, the Normal Force, and Tension

• What about the case when a projectile is shot uphill?

• Similar to the example with level ground, but now the object will land not when $y = 0$, but when it intersects the line

$$y = x \tan \theta$$
• So we want

\[ y = x \frac{\sin \varphi_o}{\cos \varphi_o} - \frac{1}{2} g \frac{x^2}{v_o^2 \cos^2 \varphi_o} = x \tan \theta \]

\[ \tan \varphi_o - \tan \theta = \frac{1}{2} g \frac{x}{v_o^2 \cos^2 \varphi_o} \]

\[ x = \frac{2v_o^2 \cos^2 \varphi_o}{g} (\tan \varphi_o - \tan \theta) \]

• Note that if \( \theta = \varphi_o \), \( x = 0 \), just as it should
Example: Throwing out a baserunner

• A third baseman needs to throw the ball to first base, 120ft away. Sometimes reporters will suggest that the ball is thrown “on a line” (e.g., with no initial vertical velocity). If so, what velocity is needed if the ball is thrown from a height of 7ft and reaches first base without bouncing?

• First, we need to find how long it would take the ball to drop 7ft:

\[
y = y_o + v_{y,o} t - \frac{1}{2} gt^2
\]

\[
0ft = 7ft + 0t - \frac{1}{2} \left(32ft/s^2\right) t^2
\]

\[
16ft/s^2 t^2 = 7ft
\]

\[
t = 0.66s
\]
• Now we look at the motion in $x$:

$$x = v_{o,x} t$$

$$120\text{ft} = v_o (0.66\text{s})$$

$$v_o = 182\frac{\text{ft}}{\text{s}} \cdot \frac{1\text{mi}}{5280\text{ft}} \cdot \frac{3600\text{s}}{1\text{hr}} = 124\text{mi/hr}$$

• That’s not humanly possible – the reporters are wrong!
Some Commonly-Encountered Forces

• We now consider other forces between objects
  – These are really all manifestations of the electromagnetic force

• One that we’ve already mentioned in passing is the *normal force*
  – This is a force of contact between two objects
  – Here “normal” means “perpendicular”, not “usual” – if one draws a tangent line to the surfaces in contact, this force will be perpendicular to that line

• Example:

What is the acceleration of the block (assuming no friction)?
• First, pick a convenient set of axes:

• Nice because we know that motion will be along the ramp
  – Since normal force is ⊥ to this, we don’t need to know it’s magnitude!

• Net (only!) force in the $x$ direction is $mgsin\theta$
  – $a = gsin\theta$
• Let’s release the block from rest, at a point on the ramp where it’s a height $h$ off the ground. How long does it take to get to the bottom, and what’s its velocity when it does?

• The block travels a distance $d = \frac{h}{\sin \theta}$
• We now apply the equations of kinematics with constant acceleration:

\[ d = \frac{1}{2} at^2 \]

\[ t = \sqrt{\frac{2d}{a}} = \sqrt{2 \frac{h}{g \sin^2 \theta}} = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}} \]

\[ v = at = g \sin \theta \cdot \frac{1}{\sin \theta} \sqrt{\frac{2h}{g}} = \sqrt{2gh} \]

• Note that the final velocity does not depend on the angle of the ramp
Tension

• A string attached to an object can also exert a force
  – But only if the string is pulled taut, and only in the direction of the string
  – This force is called tension

• We usually make the approximation that the string is massless and unstretchable, so we don’t need to worry about it sagging in the middle or about its length changing
What is the tension in the string for this situation (blocks start at rest):

- Begin by considering only the forces acting on block 1:
• The normal force and gravity cancel each other out, so the net force is \( T (= T\mathbf{i}) \)

• Therefore its acceleration is given by:

\[
a_1 = \frac{T}{m_1} = \frac{T}{m_1} \mathbf{i}
\]

• Now look at the second block:

\[
a_2 = \frac{\mathbf{F}_{\text{net}}}{m_2} = \frac{T - m_2 g}{m_2} = \frac{T - m_2 g}{m_2} \mathbf{j}
\]
• But we know that the string isn’t stretching
  – So the total length of string between the blocks is constant
  – Can only be true if the magnitude of acceleration is the same for both
• So we can set:

\[ |\mathbf{a}_1| = |\mathbf{a}_2| \]
\[ \frac{T}{m_1} = \frac{T - m_2 g}{m_2} \]
\[ T \left( \frac{1}{m_2} - \frac{1}{m_1} \right) = g \]
\[ T \left( \frac{m_1 - m_2}{m_1 m_2} \right) = g \]
\[ T = \frac{gm_1 m_2}{m_1 + m_2} \]