Lecture 15: Fermat’s Principle, and Image Formation

• One of the goals of physics is to explain many phenomena starting from a few fundamental principles
  – Newton’s Law of Gravity, for example, explained both the motion of the planets and the falling of apples on Earth
• Fermat came up with a principle that explained the propagation, reflection and refraction of light:
  – light always travels in the path that minimizes the time it takes to get from point A to point B
• This clearly is consistent with the fact that light travels in straight lines within a given medium
• The text shows how Snell’s Law arises from Fermat’s principle
Reflection According to Fermat

• Let’s say that a set of light rays start at point A
  – where will the ones that end up at point B hit the mirror?

\[ A (x_A, y_A) \]
\[ B (x_B, y_B) \]

Mirror at \( x = 0 \)

Need to find \( y_m \) that minimizes total distance traveled

\[ d = d_A + d_B \]
\[ = \sqrt{x_A^2 + (y_m - y_A)^2} + \sqrt{x_B^2 + (y_B - y_m)^2} \]
• To find the minimum, we need to take the derivative with respect to $y_m$ and set it equal to 0:

$$\frac{dd}{dy_m} = \frac{y_m - y_A}{\sqrt{x_A^2 + (y_m - y_A)^2}} + \frac{y_B - y_m}{\sqrt{x_B^2 + (y_B - y_m)^2}} = 0$$

$$\frac{y_m - y_A}{\sqrt{x_A^2 + (y_m - y_A)^2}} = \frac{y_B - y_m}{\sqrt{x_B^2 + (y_B - y_m)^2}}$$

• From the figure, we see that this means:

$$\sin \theta_1 = \sin \theta_2$$

$$\theta_1 = \theta_2$$

exactly as we expected!
The next topic we’ll explore is the formation of images – images occur when light appears to originate in some place different from the object that reflected the light.

Probably the most common image is that formed by a flat mirror – when we stand in front of the mirror, it appears to us that there’s a copy of us standing behind the mirror.

Here’s how it works:

- Light rays that enter the eye appear to “trace back” to the image.
Size and position of the image

- We’ll see many more examples of image formation
  - both from mirrors with different shapes and from lenses
- We want to determine the size and position of an image
- We can do this by considering only a few of the rays of light coming from the object and forming the image
• From the diagram on the previous slide we learn the following about the image formed by a flat mirror:
  – the image is always behind the mirror. Such images are called *virtual images*
  – the image has the same size as the object
  – the image is upright (arrow points upward for both object and image)
  – the image appears the same distance behind the mirror as the object is in front of it \((p = q)\)

• Note that an image (even a virtual one) has a definite size and position
  – i.e. these quantities don’t depend on the position of the person observing the image
Images formed by spherical mirrors

- By changing the geometry of the mirror, we can alter the behavior of images
- One example is a mirror formed into the shape of a sphere
- Let’s first consider a concave spherical mirror (this means the mirror is on the inner surface of the sphere)

- Key fact: parallel light rays close to the principal axis reflect through a common point
  - This is the *focal point* of the mirror
Ray diagrams

- Imagine we have an object with one end on the principal axis
  - where does the image formed by the spherical mirror appear?
- Find out by drawing three rays from the tip of the object:
  1. Parallel to the principal axis, reflects through focus
  2. Through focus, reflects parallel to principal axis
  3. Through center, reflects back on itself
- Reflected rays meet at tip of image!
• We really only need two rays – the third acts as a cross-check
• As with the flat mirror, we’ll define some distances:

![diagram](image)

• Note that I snuck in an “extra” ray here (the green one)
  – From this we see that:

\[
\frac{p}{h} = -\frac{q}{h'}
\]

\[
\frac{q}{p} = -\frac{h'}{h} = M
\]

**Magnification.** The – sign means the image is upside-down
• The yellow triangles on the previous slide are similar, so we know that:

\[
\frac{h}{p - R} = -\frac{h'}{R - q}
\]

\[
h' = \frac{q - R}{h} = -\frac{q}{p}
\]

\[
pq - pR = qR - pq
\]

\[
1 - 1 = 1 - 1
\]

\[
\frac{1}{R} - \frac{1}{q} = \frac{1}{p} - \frac{1}{R}
\]

• This leads to the **mirror equation**:

\[
\frac{1}{p} + \frac{1}{q} = \frac{2}{R}
\]

• For the spherical mirror it happens that \(f = R/2\). The general form of the mirror equation is:

\[
\frac{1}{p} + \frac{1}{q} = \frac{1}{f}
\]
Sign Conventions

• Clearly $p$ is always a positive number
• As I drew the diagram, $q$ was also positive
  – And the image was *real*, not virtual like the one from a flat mirror
• But in some cases the mirror equation will tell us that $q$ is negative
  – This means the image is *behind* the mirror (and is therefore virtual)
  – This happens when $p < f$
  – Note that the image will be upright in this case
• The equation also holds when $R$ is negative
  – This means the mirror is convex rather than concave