Single Particle Mechanics, According to Newton

- Now that we’ve gotten the mathematical tools down, we can begin to explore the physics of classical mechanics
- Sir Isaac Newton was first to formulate laws of mechanics, as follows:
  
  I. A body remains at rest, or moving at constant velocity, unless acted upon by a force
  II. When a force does act on a body, the rate of change of momentum is equal to the force
  III. When two bodies exert forces on each other, the forces are equal in magnitude and opposite in direction

And that’s “all” there is to mechanics
• Note that Newton’s Three Laws require an understanding of two quantities – force and momentum – that are not otherwise defined
  – They also require an understanding of concepts such as position and time, but those definitions are obvious
  • …or at least we thought they were – quantum mechanics and relativity have shown otherwise
  – We define momentum as \( mv \), leaving mass as the quantity without an independent definition
• Therefore, one can think of Newton’s statements as two definitions, plus one physical law
  – The whole issue of which quantities we should consider “primitive” (that is, able to be defined without using any particular laws of physics) is a non-trivial one
Definitions and Laws

• As an example of possible interpretations, Thornton and Marion take mass to be a primitive quantity, and use Newton’s first two laws to define force

• Others (see Symon) take force to be a primitive quantity, and use Newton’s laws to define mass

• Both are perfectly reasonable interpretations, and the result of any calculation won’t depend in which one you prefer
The Key Role of Differential Equations in Mechanics

• A common situation in mechanics is the following:
  – we know the forces that act on an object, and wish to determine the object’s position as a function of time

• Newton’s second law is nothing other than a second-order differential equation relating these quantities:

\[ m\ddot{r} = F \]

• So knowing how to solve such equations is essential for doing mechanics

• In the era when mechanics was being developed, finding such solutions was a major effort
  – Today there are computer programs that do it for you
  – It’s still our responsibility to find the correct equation of motion
Reference Frames

• In order to define position, velocity, and acceleration, one needs to specify a *frame of reference*
  – That is, a coordinate system with a specified origin and axis orientation
• However, Newton’s Laws are not valid in all coordinate systems!
  – Imagine an experiment done on a spaceship in interstellar space
  – Take the origin of the coordinate system to be fixed to the ship
  – Assume there’s a ball floating at rest in the ship…and now fire the rockets

The ball accelerates in the \(-x_2\) direction, even though no force acts on it
Inertial Frames

• Those special frames in which Newton’s Laws actually work are called *inertial* frames.

• At this point, it may seem that Newton is vastly overrated — i.e., his laws only work in the reference frames in which they work!

• But let’s assume one has found an inertial frame
  – Now create a second frame, moving at constant velocity with respect to the first one
  – Let \( \mathbf{r} \) be a particle’s position in the original (inertial) frame, and \( \mathbf{r}' \) be its position in the new frame.

\[
\mathbf{r} + \delta = \mathbf{r}'
\]
Now consider the velocity and acceleration of the object in each frame:

\[ \mathbf{v} = \dot{\mathbf{r}} \quad \mathbf{v}' = \dot{\mathbf{r}}' = \dot{\mathbf{r}} + \ddot{\mathbf{\delta}} \]
\[ \mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} \quad \mathbf{a}' = \dot{\mathbf{v}}' = \ddot{\mathbf{r}} + \ddot{\mathbf{\delta}} \]

But since the two frames are moving at constant velocity with respect to each other, \( \ddot{\mathbf{\delta}} = 0 \). This means that:

\[ \mathbf{a}' = \ddot{\mathbf{r}} = \mathbf{a} \]

Therefore, Newton’s Laws are valid in the new frame as well, so it’s also an inertial frame.

Once an inertial frame is found, any frame moving at constant velocity with respect to it must also be an inertial frame.
A Note on the Third Law

• Imagine a situation where forces are not transmitted instantly between two bodies, but rather propagate at some velocity $c$
  – This is true for any real force in nature

• Assume two bodies are at rest, exerting attractive forces on each other

• Now assume body 1 moves instantaneously to a new position
  – We know this can’t really happen without an infinite force being applied, but take it as an approximation
• Before a time $t = d/c$ has passed, the forces now look like this:

Body 2 has yet to “notice” that body 1 has moved. Since body 1 is moving through the static force field generated by 2, it always feels a force toward 2.

• The two forces are no longer equal or opposite
• Newton’s Third Law really only works in static situations
  – A hint that there’s more to physics than just Newton!
• Since $c$ is large compared to the velocities of the objects we’ll be considering, we usually take Newton’s Third Law as a good approximation
Motion of a Single Particle

• Now we apply Newton’s Laws to the motion of a single particle

• Procedure for solving problems is:
  1. Make a diagram indicating all forces acting on the object
  2. Set up a convenient coordinate system
     • e.g., if the motion is along a line, choose one axis to lie along that line
  3. Use Newton’s Laws to determine each component of the acceleration
  4. Solve the differential equation to determine velocity and position
     • During this step, one takes into account the initial conditions, such as initial position or velocity
Example 1: Constant forces

- If the force acting on a particle is constant, the equation of motion is:
  
  \[ m \ddot{r} = F \]

- In this case, we can always choose one of the axes (say \( x_1 \)) to lie in the direction of the acceleration:
  
  \[ m \ddot{x}_1 = F = m \frac{dv_1}{dt} \]

- This differential equation is easy to solve by integration:
  
  \[ F dt = m dv_1 \]
  
  \[ v_1(t) = \frac{F}{m} t + v_{1,0} = v_{1,0} + at \]
• One more integration gives us the position as a function of time:

\[
\frac{dx_1}{dt} = v_{1,o} + at
\]

\[
dx_1 = \left( \frac{F}{m} t + v_{1,o} \right) dt
\]

\[
x_1(t) = x_{1,o} + v_{1,o} t + \frac{1}{2} at^2
\]

$v_{1,o}$ and $x_{1,o}$ are constants determined by the initial conditions of the problem.