Physics 321
Theoretical Mechanics I

University of Arizona
Fall 2004
Prof. Erich W. Varnes
Administrative Matters

• Contacting me
  – I will hold office hours on Tuesday from 1-3 pm
  – Room 420K in the PAS building
  – I am also available by appointment
    • Phone: 626-0217
    • E-mail: varnes@physics.arizona.edu
  – Course web page
    • http://www.physics.arizona.edu/~varnes/Teaching/321Fall2004
      – Accessible through StudentLink web pages
    • Homework sets and solutions, exam solutions, and course
      information will be posted here
    • Also includes form to provide anonymous feedback about the course
Requirements and Expectations

• Your responsibilities for the course are:
  – Homework:
    • Assignments will be handed out on Monday, and due the Wednesday of the following week
    • Total homework score represents 25% of your final grade
  – Homework tips and suggestions
    • Doing the homework is critical to learning the material in this course
    • Many of the problems will be difficult. Feel free to discuss them and share ideas with your classmates. Working together can be a very effective way of learning.
      – However, to write up your own solutions (no copying!) and make sure you understand the problem.
    • If you get stuck, come see me (or send e-mail)
• Midterm exams:
  – There will be four midterm exams given during the class lecture period
  – Your lowest score will be dropped
  – The remaining three exams will each count for 10% of your final grade

• Final exam:
  – This will be a two-hour exam
  – Will cover material from the entire course
  – Will be worth 30% of your final grade
Some of you are enrolled in the honors section (P321H)
For the honors section, the homework and exams will total 90% of the final grade
Remaining 10% earned by writing original review article on problem of current interest in classical mechanics
  - Pick any topic you like (but I must approve it)
    • I have a list of potential topics to choose from
  - Article must review one or more recent journal articles describing research on the topic, and demonstrate a clear understanding of the problem and the research in the paper
Prerequisites

• You should have already taken the following courses (or their equivalents at another university):
  – Physics 241H or 251 (Introductory Electricity and Magnetism)
  – Mathematics 223 (Vector Calculus)
• In addition, you should have already taken, or currently be enrolled in, Mathematics 254 (Ordinary Differential Equations)

If you do not have these prerequisites, I recommend that you discuss your situation with me
Why Study Mechanics?

• You have all had at least one semester of mechanics already. In particular, it should come as no surprise to you that the equation of motion we’ll be studying all semester long is:

\[ \mathbf{F} = m \mathbf{a} \]

• So in that sense, there’s nothing new here
• You also already know that Newton’s “Laws” of physics aren’t really exact. They break down when applied to situations with large velocities or small distances
  – Relativity and quantum mechanics represent (as far as we know) the real way nature behaves
So at first glance, it appears we’ll be going over stuff you already know, and isn’t correct physics anyway

So why bother? There are a few good reasons:

1. Much of mechanics is not covered in a first-year course
   - If you look back in your freshman texts, you’ll see that only simple situations are discussed
   - For more complicated situations (even slightly more complicated) directly applying Newton’s Laws becomes cumbersome
   - Thus, some completely new techniques, and new ways to formulate the laws of mechanics, will be introduced

2. These new formulations are more closely related to quantum mechanics
   - No coincidence; those who developed Q.M. were well trained in classical mechanics.
3. Chance to learn and practice a lot of mathematics
   - Will help in future courses and your career

4. It’s fun physics
   - Unlike Q.M. and relativity, you can see classical mechanics in action every day
   - Despite this familiarity, we’ll probably find a few interesting surprises along the way
Single Particle Mechanics, According to Newton

- Sir Isaac Newton was to first to formulate a complete set of laws of mechanics, as follows:
  
  I. A body remains at rest, or moving at constant velocity, unless acted upon by a force
  
  II. When a force does act on a body, the rate of change of momentum is equal to the force
  
  III. When two bodies exert forces on each other, the forces are equal in magnitude and opposite in direction

And that’s “all” there is to mechanics
• Note that Newton’s Three Laws require an understanding of two quantities – force and momentum – that are not otherwise defined
  – They also require an understanding of concepts such as position and time, but those definitions are obvious
    • …or at least we thought they were – quantum mechanics and relativity have shown otherwise
  – We define momentum as $mv$, leaving mass and force as the quantities without an independent definition
• Therefore, one can think of Newton’s last two statements as one definition, plus one physical law
  – The whole issue of which quantities we should consider “primitive” (that is, able to be defined without using any particular laws of physics) is a non-trivial one
Definitions and Laws

• As an example of possible interpretations, Thornton and Marion take mass to be a primitive quantity, and use Newton’s second law to define force.

• Others (see Symon) take force to be a primitive quantity, and use Newton’s laws to define mass.

• Both are perfectly reasonable interpretations, and the result of any calculation won’t depend in which one you prefer.
Reference Frames

• But wait – what good is the 1\textsuperscript{st} law?
  – It seems like a special case of the 2\textsuperscript{nd} law!

• To see the answer, consider that to define position, velocity, and acceleration, one needs to specify a \textit{frame of reference}
  – That is, a coordinate system with a specified origin and axis orientation

• It seems we should be free to set up whatever coordinate system we want
  – However, Newton’s Laws are not valid in all coordinate systems!
Example of a Reference Frame Where Newton’s Laws Fail

- Imagine an experiment done on a spaceship in interstellar space
- Take the origin of the coordinate system to be fixed to the ship
- Assume there’s a ball floating at rest in the ship…and now fire the rockets

The ball accelerates in the $-x_2$ direction, even though no force acts on it
Inertial Frames

- Those special frames in which Newton’s Laws actually work are called *inertial* frames.
- At this point, it may seem that Newton is vastly overrated — i.e., his laws only work in the reference frames in which they work!
- But let’s assume one has found an inertial frame:
  - Now create a second frame, moving at constant velocity with respect to the first one:
    - We can even change the direction of the axes if we want.
  - Let \( \mathbf{r} \) be a particle’s position in the original (inertial) frame, and \( \mathbf{r}' \) be its position in the new frame.

\[
\mathbf{r} + \mathbf{\delta} = \mathbf{r}'
\]
Now consider the velocity and acceleration of the object in each frame:

\[ v = \dot{r}, \quad v' = \dot{r}' = \dot{r} + \dot{\delta} \]
\[ a = \dot{v} = \ddot{r}, \quad a' = \dot{v}' = \ddot{r} + \ddot{\delta} \]

But since the two frames are moving at constant velocity with respect to each other, \( \ddot{\delta} = 0 \). This means that:

\[ a' = \ddot{r} = a \]

Therefore, Newton’s Laws are valid in the new frame as well, so it’s also an inertial frame.

Once an inertial frame is found (using the 1st law), any frame moving at constant velocity with respect to it must also be an inertial frame – meaning all three of Newton’s Laws hold in all such frames.
The Key Role of Differential Equations in Mechanics

• A common situation in mechanics is the following:
  – we know the forces that act on an object, and wish to determine the object’s position as a function of time

• Newton’s second law is nothing other than a second-order differential equation relating these quantities:

\[ m \ddot{\textbf{r}} = \textbf{F} \]

• So knowing how to solve such equations is essential for doing mechanics

• In the era when mechanics was being developed, finding such solutions was a major effort
  – Today there are computer programs that do it for you
  – It’s still our responsibility to find the correct equation of motion