Computational Project 1: Drag

1 A reminder about doing problems in steps

We’ve been using the constant-acceleration kinematics formulas to figure out how an object’s acceleration determines how it moves.

However, we’ve done several problems where the acceleration changed from one constant to another. Consider, for instance, Homework 1 Problem 3 (the biker problem), or Exam 1 Problem 3 (the problem where a ball was dropped into water).

The appropriate approach for these problems was to solve them in steps, where the final position and velocity for the first step became the initial position and velocity for the second step.

As an example, let’s consider our bicyclist from HW1 Problem 3: “A cyclist accelerates at 1 m/s\(^2\) for 6 seconds, then travels with no acceleration for 5 seconds, then accelerates at -2 m/s\(^2\) for 3 seconds. In order to understand how to handle this situation in general, we replace all the numbers with variables. Since there are going to be a few of them, I’ll put them in a table:

<table>
<thead>
<tr>
<th>Initial position</th>
<th>(x_0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial velocity</td>
<td>(v_0)</td>
</tr>
<tr>
<td>Time after steps 1,2,3</td>
<td>(t_1, t_2, t_3)</td>
</tr>
<tr>
<td>Position after steps 1,2,3</td>
<td>(x_1, x_2, x_3)</td>
</tr>
<tr>
<td>Velocity after steps 1,2,3</td>
<td>(v_1, v_2, v_3)</td>
</tr>
<tr>
<td>Acceleration during steps 1,2,3</td>
<td>(a_1, a_2, a_3)</td>
</tr>
<tr>
<td>Length of steps 1,2,3</td>
<td>(\bar{t}_1, \bar{t}_2, \bar{t}_3)</td>
</tr>
</tbody>
</table>

The recipe is simple.

- Compute the position after each step using the position formula, using the position and velocity at the beginning of the step as the starting point:

\[
x_1 = \frac{1}{2} a_1 \bar{t}_1^2 + v_0 \bar{t}_1 + x_0
\]

\[
x_2 = \frac{1}{2} a_2 \bar{t}_2^2 + v_1 \bar{t}_1 + x_1
\]

\[
x_3 = \frac{1}{2} a_3 \bar{t}_3^2 + v_2 \bar{t}_1 + x_2
\]
• Compute the velocity after each step using the velocity formula, using the velocity at the beginning of the step as the starting point:

\[ v_1 = a_0 \bar{t}_1 + v_0 \]  
\[ v_2 = a_1 \bar{t}_1 + v_1 \]  
\[ v_3 = a_2 \bar{t}_1 + v_2 \]  

• Finally, compute the time after each step by just adding the time it took:

\[ t_1 = t_0 + \bar{t}_1 \]  
\[ t_2 = t_1 + \bar{t}_2 \]  
\[ t_3 = t_2 + \bar{t}_3 \]  

2 An illustration: Solving the biker problem in a table

Complete the table, calculating the position and velocity after each step:

<table>
<thead>
<tr>
<th>n</th>
<th>( t_n ) (s)</th>
<th>( t_n ) (s)</th>
<th>( x_n ) (m)</th>
<th>( v_n ) (m/s)</th>
<th>( a_n ) (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>6</td>
<td>18</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0</td>
<td></td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3 Warmup: Free fall

For your homework, I gave you a problem with three steps. The power of using a computer is that it can handle 10,000 steps in the blink of an eye, so let’s figure out how to do that. Essentially, we’re just going to use a spreadsheet to fill out a table, just like you did by hand above.

As a warmup, consider a freely-falling object where the acceleration is always \(-g\). We don’t need to use multiple steps here since \(a\) is constant, but it’s a good way to learn the technique.
Open the spreadsheet `freelfall-template.xlsx`, available on the course website.

Note that the calculations for the first row, cells C6 to F6, are already filled out for you. A few things to observe about the formulas typed into the cells:

- All the formulas begin with an equals sign: this means “the value in this cell equals the result of the following...”
- You can refer to the values in other cells by typing their location: for instance, notice that the formula in cell D6 refers to cell D5 (the previous position), cell F6 (the current acceleration), and cell E5 (the previous velocity).
- You can also name cells and refer to them by name: I've named cell H5 “g” and named cell H8, holding the length of each step, “step”.
- Whenever you change a variable or a formula, everything recalculates itself.

Play with the values for \( g \) and the step length and verify that the calculations match the expected results from your kinematics formulas.

Now, highlight a large square of cells, with the values c6-f6 at the top (for instance, a square from C6 to G50), and either choose “Fill Down” from the menu or hit Control-D. Notice:

- The spreadsheet copied all the formulas from the top row down for you
- All of the cell names in the formulas were automatically modified, so that each row refers to the previous row
- The graphs in the spreadsheet automatically updated to plot position and velocity for you

Play around with the values of \( g \), \( v_0 \), and the step length and see how they affect the plots.

**Task 1:** Once you’ve got this working, call your TA or coach over to check your work. (2 points)

### 4 The drag force

Free-fall is something we know how to handle with pencil and paper, but what about an object falling under the influence of drag?
In one dimension, the drag force on an object can be approximated as

\[ F_d = -\gamma v, \]

where \( \gamma \) is a “drag constant” that depends on the object’s shape and size.

This immediately poses a problem, since motion in the presence of drag isn’t constant acceleration: as the object speeds up or slows down, the drag force will change, and thus so will its acceleration.

One way to get a pretty good result for this is to use the “step” approach. Even though the acceleration’s not constant, it’s close enough to constant if we make our steps small enough. We can use the previous step’s velocity to calculate the drag force in the current step; this is an approximation, but if the steps are small enough it works out well enough.

Thus, the only modification we need in order to make our spreadsheet calculate how an object with drag falls is to change the acceleration formula. The business of doing the kinematics in steps will automatically take care of the math for us.

Task 2: If an object experiences a drag force \( F_d = -\gamma v \) as it falls, its acceleration is now no longer just \( g \). What is it? Write a formula for its acceleration and call your TA or coach over to check it. (2 points)

5 Answering some questions...

Put this acceleration formula into your spreadsheet. Note that you can write “mass” for the object’s mass, and “dragconstant” for the drag constant, since I’ve named cells for you to put these quantities in.

Task 3: Use your spreadsheet to answer the following questions, and call your TA or coach over to check your answers.

1. A baseball falls out of a helicopter flying a kilometer in the air. How long does it take to hit the ground? A baseball has a mass of 150 grams and a drag constant around 0.03 N*s/m. Show your TA or coach a plot of the baseball’s position vs. time. (2 points)
2. A tennis ball (mass 60 grams, drag constant 0.05 N*s/m) is thrown straight up at a speed of 40 m/s. How long does it take to get to its highest point? From there, how much longer does it take to come back to the ground? How did you determine this? (2 points)

3. Play with the mass and drag constant of your falling objects. Explain why a light object seems to fall more slowly than a heavy one. (2 points)

4. Now, drop your tennis ball out of a helicopter. Examine the position vs. time and velocity vs. time graphs for a period of at least 10 seconds. Discuss why they look the way that they do, based on the form of the drag force. (2 points)